

RESEARCH ARTICLE | AUGUST 20 2026

Causality analysis of vacuum chamber facility effects for Hall effect thruster operation

Special Collection: [Impact of Vacuum Facilities on Plasma Thruster Operation](#)

Seth J. Thompson ; Kentaro Hara ; Janice D. Cabrera ; Mitchell L. R. Walker ; John D. Williams 



J. Appl. Phys. 140, 073305 (2026)

<https://doi.org/10.1063/5.0325466>



Articles You May Be Interested In

The effect of facility background pressure on hollow cathode operation

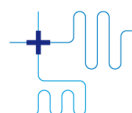
J. Appl. Phys. (September 2021)

Magnetic field tailoring effects on ion beam properties in cylindrical Hall thrusters

J. Appl. Phys. (January 2022)

Effect of body bias on near-field electron behavior in a magnetically shielded Hall thruster

J. Appl. Phys. (August 2026)



Boost Your Lab's Performance
in Quantum Research



[Find out more](#)

We support your application with

- Performant instrumentation
- Intuitive software
- In-depth application support

Causality analysis of vacuum chamber facility effects for Hall effect thruster operation

Cite as: J. Appl. Phys. **140**, 073305 (2026); doi: [10.1063/5.0325466](https://doi.org/10.1063/5.0325466)

Submitted: 30 January 2026 · Accepted: 29 June 2026 ·

Published Online: 20 August 2026



Seth J. Thompson,^{1,a)} Kentaro Hara,² Janice D. Cabrera,³ Mitchell L. R. Walker,³ and John D. Williams⁴

AFFILIATIONS

¹Department of Mechanical Engineering, Colorado State University, Fort Collins, Colorado 80523, USA

²Department of Aeronautics and Astronautics, Stanford University, Stanford, California 94305, USA

³School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, Georgia 30332, USA

⁴Department of Mechanical Engineering, Colorado State University, Fort Collins, Colorado 80523, USA

Note: This paper is part of the Special Topic on Impact of Vacuum Facilities on Plasma Thruster Operation.

^{a)}Author to whom correspondence should be addressed: seth.thompson@colostate.edu

ABSTRACT

Characterization of plasma dynamics is critical for controlling electric propulsion systems on spacecraft. Understanding the causal relations between measurement data can play an important role both in minimizing the number of diagnostics and in informing which dynamic information needs to be accounted for in a computational model. In this paper, we apply extended convergent cross mapping (eCCM) to three measured telemetry signals—cathode current, discharge voltage, and cathode-to-ground voltage—collected from a krypton-fed magnetically shielded Hall-effect thruster with base operating pressures of 6.8 and 8.5 μTorr at the 4.5 and 6 kW power levels, respectively. Data were also collected at elevated pressures, two to three times the base operating pressure. We discuss the sensitivity of the cross-mapping skill to the embedding parameters and background pressure in the vacuum chamber. We propose eCCM as a framework for characterizing dynamic relationships in the Hall-effect thruster circuit, guiding the selection of signals and the prioritization of dynamic measurements in subsequent physics-based modeling and diagnostic efforts. The results show that cross-mapping skill decreases across nearly all signal pairs for both power levels, indicating a reduction in mutual dynamic information between electrical circuit components as background pressure increases. The analysis suggests that the discharge plasma is coupled to other unmeasured components in the system as the facility pressure varies.

© 2026 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>). <https://doi.org/10.1063/5.0325466>

I. INTRODUCTION

Hall-effect thrusters (HETs) are considered one of the primary electric propulsion (EP) devices of choice for many satellites, selected for use in low Earth orbit commercial constellations and for discovery-class deep-space missions.¹ Despite their growing use, fundamental gaps in understanding physical phenomena still limit our ability to deploy predictive codes as design tools or to predict on-orbit performance.² One major challenge for on-orbit performance prediction is the need to test the devices in ground facilities under conditions that differ from those in space. These differences take the form of a finite background pressure in vacuum chambers, orders of magnitude higher than on-orbit conditions;^{3–6} electrically

conductive walls that intercept the thruster plasma plume;^{7–9} and significant deposition of vacuum wall material onto thruster components due to sputtering of the facility walls.^{10–12} Attempts to address the effect of facility pressure on thruster operation often rely on parametric sweeps of background pressures across a range of achievable pressures in ground-test facilities.^{3–6} However, the HET discharge is a highly nonlinear process, and it remains unclear whether these trends accurately capture the information needed to predict on-orbit operation. Furthermore, these studies often evaluate steady-state performance or measurements, despite observations that key model parameters, such as anomalous electron mobility, may be time-dependent.^{13,14} Apart from elevated chamber pressure, these facility studies are further complicated by the effects that the

conductive walls of vacuum chambers can have on thruster operation.^{7–9} To address uncertainties arising from testing these devices in imperfect ground facilities with dynamically varying, non-linear model parameters, data-driven state-estimation approaches that use test data to inform key model parameters have been explored to capture thruster-specific behavior.^{14–16} However, these data-driven models that capture dynamic behavior require relevant dynamic data to most effectively reduce uncertainty in parameter estimation. Determining which time-varying signal or signals are most relevant is crucial for improving model fidelity and performance.

Time-resolved measurements of direct plasma properties, such as species density, electron energy distribution, ion and neutral velocity distributions, and plasma potential, are ideal as inputs for data-driven models. However, measuring these properties directly in the spatial or temporal scales of interest is difficult and not readily scalable. Even when time-resolved measurements can be made, determining whether the dynamics exhibited in one property or spatial location are dynamically relevant to the thruster behavior or specific model physics can remain elusive. Therefore, selecting accessible time-resolved measurement datasets that maximize information gained in data-driven thruster models requires additional scrutiny to ensure that the signal is dynamically relevant to the system and can be incorporated into the model. Although models can include several streams of measurement data, it is just as critical to avoid irrelevant signals that have no bearing on the device's operation.

To aid in selecting which signals to include in data-driven and physics-based modeling frameworks, we deploy a dynamic causality-determination technique, extended convergent cross mapping (eCCM).¹⁷ This technique can be used to infer causal relationships between dynamic signals, providing context for selecting data streams. This, in turn, can be used to exclude specific signals or adjust models to account for the most relevant sensor signals. Signals with strong causal links will provide more unique information, thereby improving uncertainty reduction in key model parameters. These time-varying signals can be either high-speed diagnostic measurements or periodic measurements when multi-modal data-driven frameworks are used.¹⁶ When evaluating accessibility and scalability, the telemetry signals, such as discharge voltage and current measurements, are among the top candidates. These signals are readily accessible, relevant to the entire thruster system (e.g., discharge, cathode, and near-field), and can be monitored non-invasively. Previous data-driven frameworks have used applied power and discharge currents for data-driven models that relate bulk plasma parameters to input power or current.^{14,16,18} Adapting these models to include additional signals, such as discharge voltage, cathode-to-ground voltage, the potential of floating mounting structures, thruster-body potentials, or other accessible circuit elements, may better capture state evolution if shown to have a strong causal relationship. The focus of this work is the continued development and application of the eCCM framework as a tool for characterizing dynamic systems, such as the Hall-effect thruster discharge, with the goal of identifying key dynamic relationships to inform future modeling and measurement campaigns. We do not seek to extract physics-based conclusions about the discharge directly from the three telemetry signals considered here; doing so rigorously would require co-measured dynamic

plasma property data or coupling to a physics-based model, both of which are left to future work. Instead, we use eCCM to quantify which signal pairs carry unique, causally coupled dynamic information under varying facility conditions, and to evaluate how that information evolves with background pressure.

In considering the use of dynamic signals of interest, it is critical to understand what dynamic information the measurements capture. Hall-effect thruster discharges exhibit a plethora of oscillations and instabilities across a wide range of spatial and temporal scales. The relevant timescales span several orders of magnitude, from nanoseconds for electron-cyclotron frequencies to hundreds of microseconds for ionization-related oscillations, and range in spatial dimensions from fractions of a millimeter for Debye lengths to fractions of meters for the device size. Many of these discharge oscillations were reviewed by Choueiri in 2001,¹⁹ and these oscillations remain areas of active research.² Breathing-mode oscillations have remained of top interest due to their influence on stability and performance,^{20–22} while kinetic instabilities, such as electron-cyclotron drift instabilities, remain of keen interest due to their suspected influence on anomalous electron transport.^{23,24} Furthermore, the forced coupling of the HET discharge to a hollow cathode discharge through a common DC circuit means that oscillations native to hollow cathode operation^{25–28} may manifest in global circuit measurements of a thruster. Beyond plasma-specific dynamics, slow transient behavior during device operation is observed, ranging from tens of seconds to hours, due to thermal effects and material outgassing.^{10,29,30} This combination of many non-linear coupled dynamics that manifest in the telemetry makes it challenging to determine the most relevant timescales that capture all important dynamics in the system. For our work, we will focus on dynamics in the 5 kHz–1 MHz range, which encompasses most of the oscillations that capture the state of the plasma dynamics, with the potential exclusion of high-frequency kinetic instabilities.

Further complicating the evaluation of useful dynamic signals for HET modeling is the effect of environmental parameters on the underlying dynamic system. Identified causal relationships may be influenced by external environmental parameters that alter the underlying system dynamics. For example, elevated background pressure resulting from limited pumping speed can alter charge-exchange plumes and modify expected neutral particle profiles near the thruster,^{31,32} thereby changing the relationships among dynamic properties in the discharge or near-field region. Some studies have demonstrated pressure dependence of the electric field and electron temperature in magnetically shielded thrusters.³¹ In addition to the facility-grounded surfaces and background pressures, questions remain about the effects of thruster-body potential on thruster operation⁶ and the constant redeposition of wall materials,¹² which may affect the isolation of that potential over the length of a test. As a result, causal relationships identified in one environment may vary from facility to facility, across a range of pressures within a single facility, or even over a set period within a single facility and under a given pressure.

We emphasize that the present work is not intended to directly provide a predictive tool for extrapolating thruster behavior from ground-test pressures to on-orbit conditions. No causal relationship predictor exists that can be confidently applied

outside the measured pressure range. Rather, the objective here is to establish an empirical, data-driven characterization of how the causal relationships among key telemetry signals evolve with background pressure, thereby providing a foundation for constructing and validating future dynamic, physics-based models that would eventually enable principled extrapolation.

In this study, we apply eCCM to high-speed telemetry signals from an H9 thruster, including cathode current, I_{cath} ; discharge voltage, V_{dis} ; and cathode-to-ground voltage, V_{cg} , for different vacuum chamber pressures. We discuss how these data were collected in Sec. II, followed by a description of the eCCM process and the related reconstruction parameters and calibration processes needed to identify causal relationships and their associated delay responses in Sec. III. We apply the analysis technique and discuss the implications of selecting appropriate embedding parameters for thruster operation at 4.5 and 6 kW across several pressures and discuss the results of signal selection and the implications for further applications of causal analysis in data-driven frameworks in Sec. IV.

II. HALL-EFFECT THRUSTER OPERATION

Extended convergent cross mapping is applied to HET telemetry data collected in a previous facility-pressure-effect test on thruster impedance, as presented in Ref. 32. This test was conducted with an H9 magnetically shielded HET developed by the University of Michigan, Air Force Research Laboratory, and Jet Propulsion Laboratory³³ and operated on krypton in VTF-2 at the Georgia Tech High-power Electric Propulsion Laboratory (HPEPL). The centerline LaB₆ hollow cathode was operated at a fixed cathode flow fraction of 7% across all power conditions. The lowest operational pressure for the thruster was 6.8×10^{-6} Torr at the 4.5 kW operating condition and 8.5×10^{-6} Torr at the 6 kW operating condition. The anode and cathode flow were adjusted to maintain constant power at elevated pressures. Telemetry data were captured with a Teledyne LeCroy Digital Oscilloscope HDO6104, using Powertek DP-25 differential voltage probes (bandwidth of 25 MHz) for both discharge and cathode-to-ground voltage measurements. Cathode current was measured with a Teledyne LeCroy CP150 current clamp (bandwidth of 10 MHz). Data from the oscilloscope were collected at 10 MHz. The thruster was operated with an inline RC filter to protect the power supply from reflected oscillations, with a resistance of 0.533Ω and a capacitance of $100 \mu\text{F}$.⁹ The thruster body was tied to the chamber ground for this test, matching the parent test campaign of Cabrera *et al.*³² Community practice regarding thruster-body grounding varies, though floating or cathode-tied thruster-body configurations may be more common in recent qualification tests;³⁴ however, development efforts have often evaluated floating and grounded-body configurations in early thruster characterizations.³⁵ To adjust chamber pressure, krypton gas was injected through a $\frac{1}{4}$ in. Swagelok connection located 1.8 m axially downstream of the thruster face and 1 m below the thruster centerline with the outlet directed toward the chamber wall to attempt to follow recommended practice of Dankanich *et al.*³⁶ and was selected in the pressure-study configuration of Cabrera *et al.*³² both to thermalize the injected krypton against the chamber walls and to avoid

introducing a directed jet or non-uniform density field in the thruster near-field. The flow rate was adjusted until the two- and three-times baseline chamber pressures were achieved. Figure 1 shows a notional circuit diagram with depictions of the telemetry signals and gas inlet for chamber pressure adjustment. The pressure gauge, a Varian 571 Bayard-Alpert hot filament with a neutralizing inlet,³⁶ was located 1 m radially from the thruster, aligned with the thruster exit plane. The hot-filament ionization gauge produces a time-averaged pressure reading on the scale of seconds and cannot resolve fluctuations in the 5 kHz–1 MHz band targeted by this study. Chamber pressure is, therefore, treated as an environmental setpoint rather than a dynamic signal in the eCCM analysis.

The choice to focus on cathode current I_{cath} , discharge voltage V_{dis} , and cathode-to-ground voltage V_{cg} reflects three considerations. First, these three signals are externally accessible at the power-processing circuit, are routinely monitored during HET testing, and represent the class of measurements that would plausibly be available on a flight system and during qualification and acceptance testing, giving them a low barrier to practical use. Second, taken together, they sample the three principal circuit nodes of the thruster—the anode, the cathode, and the body/chamber-ground reference—and, therefore, capture dynamics associated with each of the major potential references present in a typical HET test configuration. They are not a complete set; the anode current, in particular, would also be of interest, but the present analysis draws on telemetry collected during a previously executed parent test campaign in which that signal was not recorded. Third, this class of external circuit measurement is the same class used by prior data-driven state-estimation efforts in HETs.^{14,16,37} More specific temporal diagnostics, such as in-vacuum probes, ion velocity or energy distributions, and optical emission measurements, may eventually provide richer causal information but are less practical as flight-representative telemetry and were outside the scope of the parent test campaign.³² Expansion of the analyzed signal set is identified as a direction for future work in the Conclusion.

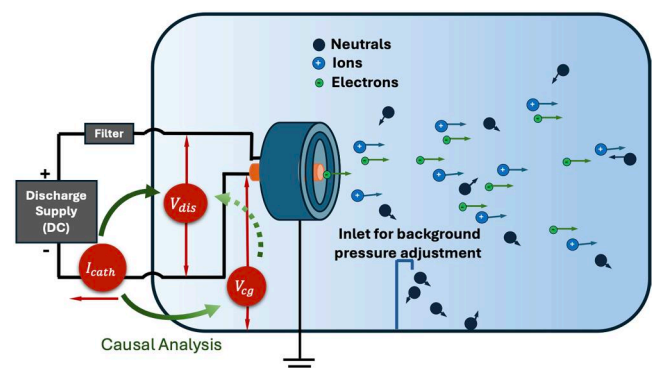


FIG. 1. Simplified H9 testing schematic with telemetry measurements used for causality analysis and notional pressure adjustment location highlighted.

III. CAUSALITY ANALYSIS

It is often the case that correlation, or the lack thereof, is used to determine if a given variable or data stream should be included in a model. When there is no noticeable change in the underlying measurement or property that correlates with the phenomena of interest, it is all too often overlooked or dropped from consideration. By evaluating the causal relationships among two or more variables, we can better determine whether the information under consideration is ultimately useful to include in a model. In other words, causal analysis can quantify the dynamic relevance of a signal used to capture the state of the system.

A. Background of extended convergent cross mapping

Granger causality has been used to evaluate causality among potentially interacting variables, focusing on improving estimates by including or excluding those variables in the prediction of a variable of interest.³⁸ This approach, however, can struggle to correctly identify causal relationships in dynamic systems where the history of one variable may be redundant due to its inseparability from other system variables, as a consequence of their causal relationship in a deterministic dynamic system.³⁹ As a result, effective inclusion and exclusion of variables cannot be achieved, leading to ambiguity in identifying causal relationships. Convergent cross mapping (CCM) addresses this challenge by fundamentally leveraging the inseparability of causally related variables in a deterministic dynamic system.⁴⁰ Furthermore, this technique is valuable where direct measurements of all underlying variables are difficult or infeasible.

Convergent cross mapping leverages Takens' theorem,³⁹ which relates a dynamic system's present states to its past states to evaluate the degree to which one variable's history is present in that of another. Sugihara *et al.* demonstrated a causal relation in a system containing dynamic variables x and y as follows. If x acts causally on y , then information about x will be present in the history of y . Additionally, reconstruction of x using the historical data of y will then be convergent to higher accuracy with the inclusion of additional data.⁴⁰ The metric used to gauge reconstruction accuracy was the Pearson correlation coefficient between the reconstructed and test signals. The reconstruction process leverages time-delay embeddings of a signal, coupled with a technique such as simplex projection,⁴¹ to perform state-space reconstruction (SSR) and produce a signal estimate of the second signal. This process can be leveraged to identify the reduced set of measurement data required to capture the state of a system if many types of measurement data can be reconstructed from a selected set of signals. The reconstruction of different measurement data streams was demonstrated using the shadow-manifold interpolation method with HET test data by Eckhardt *et al.*⁴² The technique was leveraged to generate time-resolved reconstructions of many measurement-state-based diagnostics.^{43–46} While SSR techniques are useful for time-resolved measurements, the focus of the reconstruction in CCM is on determining the nature of the relationship between the input and reconstructed signals.

By using these reconstructions to evaluate dynamic causality, it is possible to generate accurate reconstructions of a given signal from signals that were not causally related. Strong external driving variables

can dominate the dynamics of non-interacting species that appear to respond synchronously, a phenomenon in ecology known as the Moran effect.⁴⁷ Additionally, situations arise in which one variable strongly drives another to such an extent that the common dynamics of both variables lead to similar reconstruction behavior, making it difficult to distinguish true unidirectional behavior from bidirectional behavior. These situations can be broadly categorized as forms of generalized synchrony and are known to obfuscate the correct identification of causal variables.⁴⁰ To address this challenge, in 2015, Ye *et al.* developed an extension of CCM (eCCM) that considered the time shifts (note that we call these time shifts in eCCM to distinguish from the time delays used in the time-delayed phase portraits) between dynamic variables and their peak reconstruction quality to identify the dynamic response and infer directionality, i.e., whether one signal precedes or succeeds another signal, thereby improving the interpretation of CCM.¹⁷ This technique has been applied to HET simulations⁴⁸ and experimental measurements⁴⁹ and has been compared with other causality-determination techniques.⁵⁰ It is important to note that since causality is transitive, one variable's reconstruction quality may be the result of many other driving variables not measured directly. While relationships drawn between subsets of system variables must be carefully analyzed, as they can be subject to the effects of transitivity and indirect relationships, i.e., the analysis is limited to the relationships within the selected datasets, we employ causal analysis to quantify changes in telemetry characteristics across different test conditions.

B. CCM description

The premise of CCM relies on nonlinear SSR that leverages Takens' theorem.³⁹ To study if one signal $x(t)$ is causal on another signal $y(t)$, the process attempts to reconstruct the dynamic signals of the time series $x(t)$ from a delayed embedding (which is sometimes called a shadow manifold) of the time series $y(t)$. If this reconstruction can be performed with increasing accuracy, as measured by the Pearson correlation coefficient, ρ , between the reconstructed and real time series, when more data are included in the analysis, then it can be understood that x is causal on y .

The basic flow of the CCM process is outlined below. Time-resolved signals are collected synchronously for all variables of interest. The signals are split into test-train segments, i.e., $x(t = 1, \dots, N) = [x_{\text{train}}(1, \dots, N_{\text{train}}), x_{\text{test}}(N_{\text{train}} + 1, \dots, N)]$ and $y(t = 1, \dots, N) = [y_{\text{train}}(1, \dots, N_{\text{train}}), y_{\text{test}}(N_{\text{train}} + 1, \dots, N)]$, where N is the number of total time steps and N_{train} is the number of time steps for the training data. The test signals are reconstructed using the shadow manifold, which will be defined shortly, and the reconstructed signals are compared with the original test data signals to assess the reconstruction quality. The reconstruction is performed by considering a shadow manifold with a reduced number of signals: $M_y(t) = [y(t), y(t - \tau), \dots, y(t - (m - 1)\tau)]$, where τ is the time delay and m is the number of embedding dimensions. The time-delayed phase portrait was first introduced to the EP community by Greve *et al.*¹⁵ as a tool for model calibration of nonlinear dynamical systems.

At each time step of the test data signals, we take the time-delay embedding test data manifold, i.e., $M_{y,\text{test}}(t = t_0) = [y_{\text{test}}(t = t_0), y_{\text{test}}(t = t_0 - \tau), \dots, y_{\text{test}}(t = t_0 - (m - 1)\tau)]$. In addition, the

train data manifold is constructed as $M_{y,train}(t = t_1) = [y_{train}(t = t_1), y_{train}(t = t_1 - \tau), \dots, y_{train}(t = t_1 - (m-1)\tau)]$. Then, we find a finite number of t_1 values, e.g., $m+1$ values,⁴¹ of the train data manifold that result in the smallest Euclidean distance between $M_{y,test}(t_0)$ and $M_{y,train}(t_1)$. These datasets are called the near neighbors. Now, a shadow manifold of x is constructed as

$$\hat{M}_{x,test}(t_0) = \sum_{i=1}^{m+1} w_i M_{x,train}(t_i), \quad (1)$$

where $w_i = u_i / \sum_{i=1}^{m+1} u_i$ is the normalized weight, $u_i = \exp(-\frac{d_{y,i}}{d_{y,1}})$ is the weight, and $d_{y,i}$ is the Euclidean distance between $M_{x,train}(t_0)$ and $M_{x,train}(t_i)$. Note that $i=1$ is the nearest neighbor. Here, the constructed shadow manifold can be considered to have m discrete numbers such that $\hat{M}_{x,test}(t_0) = [x_{sm}(t_0), x_{sm}(t_0 - \tau), \dots, x_{sm}(t_0 - (m-1)\tau)]$, where the x_{sm} values are estimated from Eq. (1). Finally, it should be noted that the x signal can be reconstructed, which we call $\hat{x}(t)$, by taking an average over m values of $x_{sm}(t)$. The cross-mapping skill is evaluated using the Pearson correlation coefficient ρ between the reconstructed signal $\hat{x}(t)$ and the test signal $x_{test}(t)$.

C. Embedding parameter selection

As discussed in Sec. III B, embedding parameters must be selected to generate time-delay embeddings or shadow manifolds used in the cross-mapping process to reconstruct signals. Specifically, a time delay τ and an embedding dimension m must be selected. There is no mathematical requirement for the selection of τ to generate a proper embedding; however, real-world applications demonstrate that both the embedding dimension and the time-delay selection significantly impact reconstruction quality. Several methods for determining the relevant time delay τ and embedding dimension m have been developed.⁵¹ Many of these methods focus on reducing the computational cost of evaluation and on distinguishing deterministic and stochastic signals. Cao's method⁵² for the selection of the embedding dimension m , based on a modified form of identifying false nearest neighbors, has been used in many applications with thruster-relevant signals.^{49,50} However, this process requires selecting a time delay for which numerous heuristics have been proposed for given applications.⁵¹ Recently, work by Martin *et al.*⁵³ has demonstrated a robust framework that leverages the mutual information of transformed coordinates (MITCs) to select a value for τ that yielded high reconstruction quality in a noisy system. In this paper, we conduct a parametric sweep of τ and m , comparing their respective impacts on reconstruction quality for a given pair of signals. As a result, every pair of signals reconstructing one another used the embedding dimension m and the embedding time delay τ that produced the best reconstruction with no time shift l applied.

D. Time shift for eCCM

The descriptions in Secs. III A–III C describe the process for performing CCM. In eCCM, a time shift (or cross-map lag¹⁷) is used to identify directional information flow between signals. The time shift, l , is applied between the two signals used in the

reconstruction. Therefore, we perform the same CCM process, but for $y(t)$ cross-mapping $x(t+l)$. If $x(t)$ is causal on $y(t)$ with some delay, then y will have a better cross-mapping skill of previous values of x . For instance, if x acts on y after two time steps, we would expect the largest cross-mapping skill for y cross-mapping x with a shift of $l = -2$. Hence, one condition for causality is the large cross-mapping skill with a negative time shift, i.e., $l < 0$. For the same example, we can consider the other cross map, i.e., x cross-mapping y with a shift of $l = 2$. Even if the cross-mapping skill is high, indicating that a cross-mapped signal is reconstructed accurately, the positive time shift indicates that y is not causal on x . This identification of a delay in response from one variable to another is tested with an example problem used in Ye *et al.*¹⁷ and in Huerta *et al.*⁵⁰ The details of this test and the results are shown in the Appendix.

IV. RESULTS

We applied the eCCM analysis to voltage and current signals for the H9 thruster tested at the lowest achievable operating pressure (baseline), as well as twice (2×) and three times (3×) the baseline pressure at 4.5 and 6 kW operating conditions. The thruster was operated for several hours at the baseline pressure to reach steady state and avoid transient dynamics associated with thermal drift and channel outgassing.³² To evaluate the optimal embedding parameters for each cross-mapping, a parametric sweep of the embedding dimension m and time delay τ is conducted. The embedding parameters that yield the highest cross-mapping skill are then selected for the extended convergent cross mapping, in which time shifts are applied to the signals to evaluate information flow between them. For the parameter sweeps and extended convergent cross mappings, the signal data length is limited to 131 072 at a measurement frequency of 10 MHz, spanning roughly 200 breathing-mode oscillations of the H9 thruster. Hence, one time step in the measurement dataset corresponds to 100 ns. Each analysis is repeated on six such windows taken across the full ~100 ms telemetry record, and the mean reconstruction skill of the windows is shown in the following power spectral density and eCCM plots, with the standard deviation from that mean, calculated across all windows, shown as the shaded regions.

A. 4.5 kW operating condition

The lowest chamber pressure of $6.8 \mu\text{Torr}$ was achieved with the 4.5 kW operating condition. After operating for several hours to achieve thermal steady state, the telemetry was collected, and the chamber pressure was increased to twice the nominal background pressure, approximately $13 \mu\text{Torr}$. This process was repeated to achieve three times the nominal background at approximately $20 \mu\text{Torr}$. Under all conditions, the thruster was operated at a steady-state discharge voltage of 300 V and a discharge current of 15 A, the discharge current being maintained by adjusting the thruster flow rate. The 4.5 kW operating condition had the lowest peak-to-peak current oscillation relative to the mean current in the telemetry waveforms collected.

The power spectral density plots for each signal under the three background pressure conditions are shown in Figs. 2(a)–4(a). As shown in Figs. 2(a1)–4(a1), the cathode current exhibits a peak oscillation at the breathing mode around 16.4 kHz, with lower

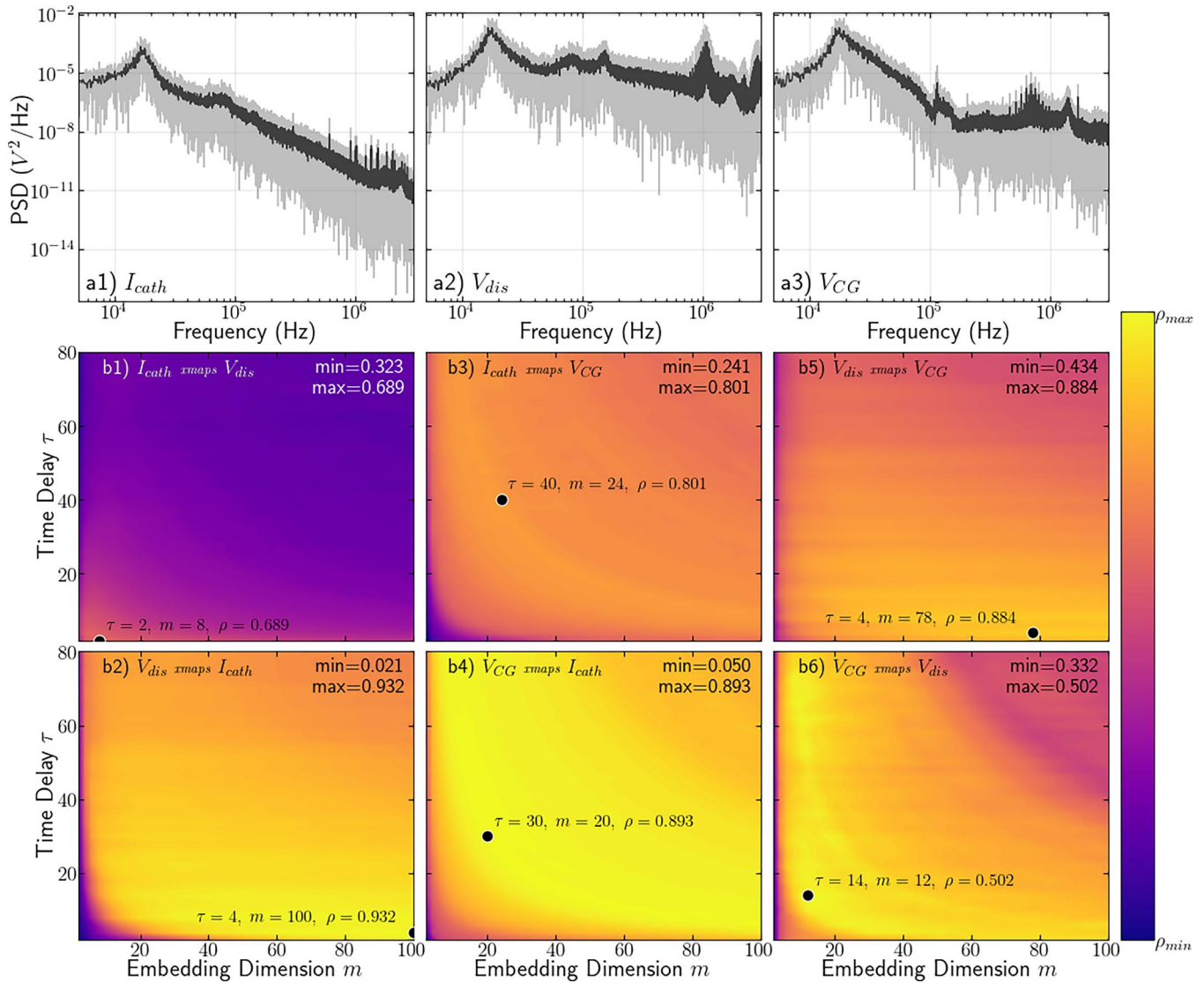


FIG. 2. Power spectral density plots [(a1)–(a3)] and embedding parameter sweep [(b1)–(b6)] of I_{cath} , V_{dis} , and V_{cg} waveforms captured from an H9 thruster operated at 4.5 kW at baseline pressure, i.e., a chamber pressure of 6.8 μ Torr.

signal content at higher frequencies. Figures 2(a2), 2(a3), 3(a2), 3(a3), 4(a2), and 4(a3) show that the discharge and cathode-to-ground voltage oscillations occur near a similar breathing-mode frequency at 17.6 kHz but with considerably more signal content in the 100 kHz–1 MHz range. In all cases, the breathing-mode peak decreased with increasing pressure, accompanied by broader peaks relative to those at the baseline operating pressure. Cathode current showed the largest change in the oscillation amplitude at the breathing-mode frequency, followed by cathode-to-ground voltage, with the smallest change in breathing-mode peak height observed in discharge voltage.

1. Parametric sweep

To perform cross-mapping of the telemetry signals, embedding parameters must be selected. Due to uncertainty in the relationship between the different dynamic signals, a parametric sweep of the embedding dimension m and time delay τ is conducted to determine the optimal embedding parameters to use for cross-mapping. The time-delay range is 2–80, and the embedding dimension is varied from 2 to 100. The parametric sweep is performed without time shifts between the signals, and the embedding parameters that yield the highest cross-mapping skill for the reconstructed signal are then used in extended convergent cross

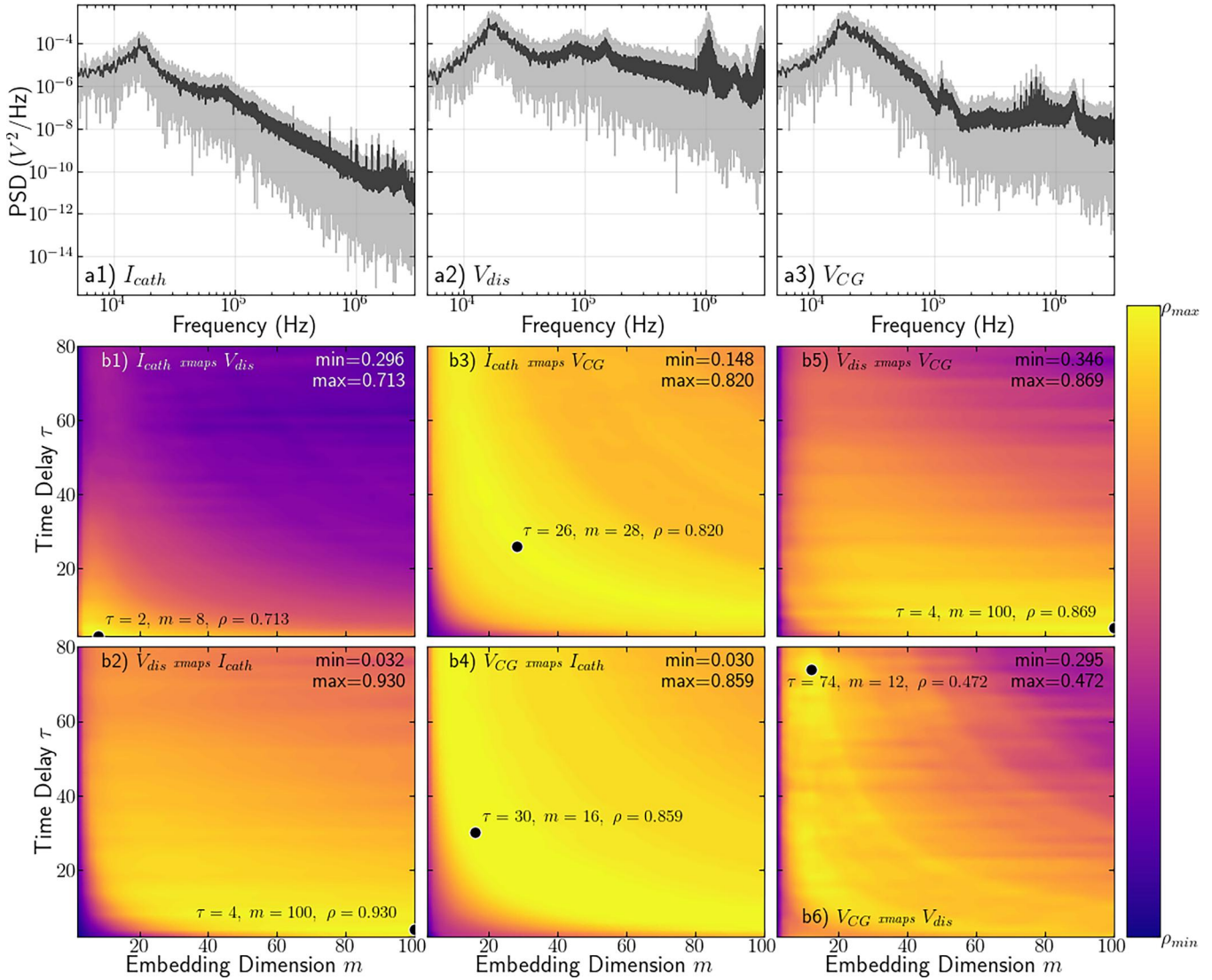


FIG. 3. Power spectral density plots [(a1)–(a3)] and embedding parameter sweep [(b1)–(b6)] of I_{cath} , V_{dis} , and V_{cg} waveforms captured from an H9 thruster operated at 4.5 kW at twice the baseline pressure, i.e., a chamber pressure of 13 μ Torr.

mapping, in which time shifts are applied to the signals. Although these signals are all measured from the same circuit operating a dynamical plasma system, there is no guarantee that the singular set of embedding parameters will yield maximum reconstruction quality for all pairs of signals used for cross-mapping.

For each signal combination, we perform two cross-mappings in each direction, i.e., I_{cath} cross-maps V_{dis} and V_{dis} cross-maps I_{cath} . The combination of an unknown relationship between the signals, measurement uncertainty, and the associated uncertainty in properly embedding each signal is why we do not assume that a single set of embedding parameters will perform well for all signals. Indeed, this can be seen in the embedding parameters that

result in the highest cross-mapping skill in Fig. 2. The cross-mapping skill of I_{cath} and V_{dis} in Figs. 2(b1) and 2(b2) shows the significant change in embedding dimensions that result in optimal reconstruction. Beyond differences observed in the directionality of cross-mapping, we observe changes in the best embedding parameters for a single signal used to cross-map different signals, as shown in the reconstructions of V_{dis} and V_{cg} by I_{cath} in Figs. 2(b1) and 2(b3), respectively. The same input signal, I_{cath} , is used for both reconstructions; however, significantly different embedding parameters are required to achieve the best cross-mapping. In general, the best embedding parameters to obtain the highest reconstruction skill vary widely across Figs. 2(b1)–2(b6).

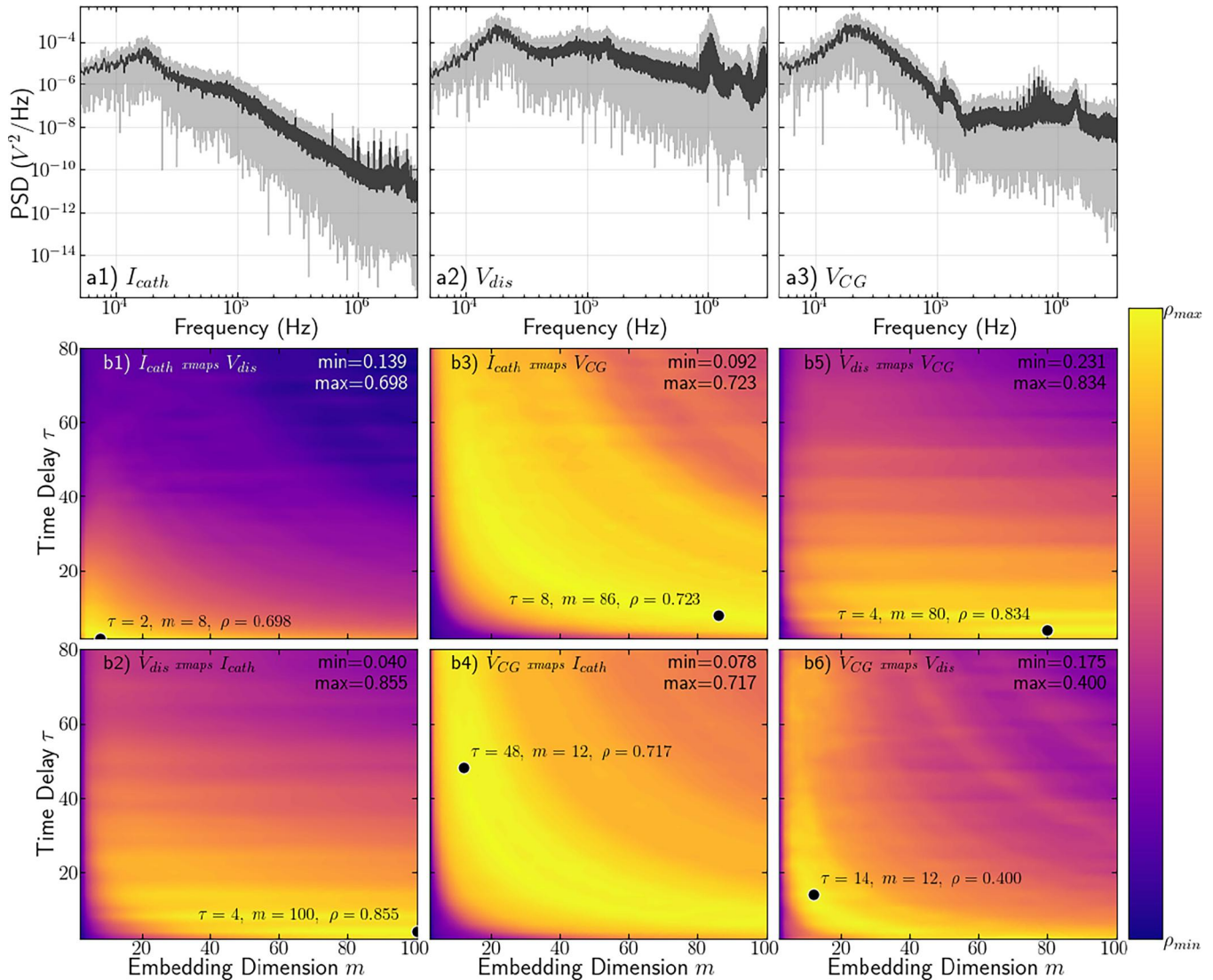


FIG. 4. Power spectral density plots [(a1)–(a3)] and embedding parameter sweep [(b1)–(b6)] of I_{cath} , V_{dis} , and V_{cg} waveforms captured from an H9 thruster operated at 4.5 kW at three times the baseline pressure, i.e., a chamber pressure of 20 μ Torr.

The power spectral density plots and parametric sweep of embedding parameters for the signals collected at two times baseline chamber pressure are shown in Fig. 3. The embedding parameters that yield the best cross-mapping skill continue to depend on the signal being cross-mapped and the directionality of the cross-mapping between the two signals. A significant increase in the optimum τ value for V_{cg} cross-mapping V_{dis} in Fig. 3(b6) is observed at the higher pressure, although the lower embedding τ in Fig. 2(b6) remains a relatively high cross-mapping skill region in both plots. This could be due to periodicity in the underlying dynamics, or to a change in the dynamics of the input or test signals, best captured by increasing the time delay for the

embeddings by about 6 μ s. There is also a slight decrease in the peak cross-mapping skill achieved in each sweep at the higher pressure.

In cross-mappings involving V_{dis} shown in Figs. 3(b2) and 3(b5), the highest cross-mapping skill was observed at $m = 100$, i.e., at the limit of the dimensional range allowed for the sweep. The effects this has on the eCCM results are discussed in Sec. IV A 2. Furthermore, additional sweeps to higher τ and m values were conducted and yielded only minor increases in cross-mapping skill. As a result, the eCCM analysis for all signals used embedding parameters that yielded the highest cross-mapping skill only within the parameter sweep range.

For a chamber pressure of $20\ \mu\text{Torr}$, corresponding to three times the base operating pressure for the thruster at $4.5\ \text{kW}$, the power spectral density plots and the parametric-sweep contours for cross-mapping are shown in Fig. 4. The continued reduction in peak height with increased broadening is observed in Figs. 4(a1)–4(a3), while the continued reduction in peak and high cross-map skill regions is observed in the contour plots in Figs. 4(b1)–4(b6). A reversion to the smaller τ value for V_{cg} cross-mapping V_{dis} was observed in Fig. 4(b6), along with a notable increase in dimension m for I_{cath} cross-mapping V_{cg} in Fig. 4(b3).

In general, the parametric sweep of embedding parameters appears justified, as the reconstructed signals and the direction of cross-mapping influenced the embedding parameters that yield the highest cross-mapping skill. In some instances, particularly for I_{cath} and V_{cg} , the relationship of the $m\tau$ product appears to produce regions of higher cross-mapping skill, as may be expected for signals of the same dynamical system, as this scalar product is related to the effective time period captured by a single embedding vector at a given time step.⁵¹ Specifically, for I_{cath} and V_{cg} , it seems to be loosely grouped around $m\tau \approx 500$, or $50\ \mu\text{s}$, which is near the breathing-mode oscillation period.

2. Time shift identification

As discussed in Sec. III A, one of the primary advantages of eCCM over previous techniques is the identification of time shifts between the causal responses of one variable to another, reflecting the information-transit time between the two signals and the direction of information flow. The process is described in detail in Sec. III B.

The difference between the resulting peaks in cross-mapping skill relative to one another, as well as the no-shift case ($l=0\ \mu\text{s}$), is used to determine whether there is a lag in the information flow between the signals. Peaks in the negative time-shift direction indicate that the reconstructed signal is causal with respect to the signal used for the reconstruction, with a delay on the order of the observed time shift. Peaks at a positive time shift indicate that the reconstructed signal is not causal on the signal used for reconstruction but does share signal dynamics. This can occur when similar external dynamics drive non-interacting variables, or when the signal performing the cross-mapping is unidirectionally driving the signal being cross-mapped. In the case of a unidirectional relationship, there should be both a positive and a negative time-shift peak in the respective causal direction. This is the classic case covered by Ye *et al.*¹⁷ Finally, as discussed in the Appendix and by Ye *et al.*,¹⁷ there are limits to the accuracy of the interpreted time shift if the time shift is less than that of the $m\tau$ product, as the shift value can be influenced by centering of the delay vector and $m\tau$ product size. We found in our example using negative-lag embeddings, following $[x(t), x(t-\tau), x(t-2\tau), \dots, x(t-(m-1)\tau)]$, that the negative shift consistently identified the correct lag value, while overpredicting in the forward-shifted peak direction, as shown in the plots in the Appendix.

Figure 5 shows the results of eCCM of the H9 telemetry signals at baseline, i.e., $6.8\ \mu\text{Torr}$ [(a1)–(a3)], $2\times$ baseline, i.e., $13\ \mu\text{Torr}$ [(b1)–(b3)], and $3\times$ baseline, i.e., $20\ \mu\text{Torr}$ [(c1)–(c3)]. As shown in the first column, Fig. 5(a1), V_{dis} has the highest cross-mapping skill of I_{cath} with a nearly instantaneous response. The

slight negative shift would indicate that V_{dis} is causally driven by I_{cath} , with little or no delay, and an observed reduction in cross-mapping skill with increasing chamber pressure, as shown in Figs. 5(b1) and 5(c1). In Fig. 5(a2), we can see that the negative peak of V_{cg} relative to I_{cath} indicates that I_{cath} is causal on V_{cg} at the baseline pressure. However, the reduction in cross-mapping skill of V_{cg} with increased pressure below that of the positive-shifted peak shown in Figs. 5(b2) and 5(c2) indicates that this may deteriorate into an indirect relationship that is still reliant on a common set of dynamics with reduced unique information about I_{cath} in the time history of V_{cg} . Finally, it can be seen in Figs. 5(a3)–5(c3) that a strong indirect relationship exists where common dynamic information in V_{dis} is also present in V_{cg} ; however, V_{cg} cannot reconstruct the dynamics of V_{dis} well. Again, the reconstructions follow the trend of the other two signal pairs, with decreasing reconstruction skill observed at higher pressures, indicating eroded mutual dynamic information.

As discussed in the parametric sweeps of embedding parameters, there are cases when the maximum embedding dimension (e.g., $m=100$) was selected as the parameter with the highest cross-mapping skill. In these cases, the signal doing the cross-mapping exceeded the cross-mapping skill in the reverse direction, leading to no change in the identified directionality or time shift l in the subsequent causality analysis. Furthermore, minor gains in the highest cross-mapping skill from expanding the parameter sweep ranges also reduce the likelihood of differences arising from sweep limitations.

We see that the discharge voltage V_{dis} appears to reconstruct the other signals most effectively; even though the positive peak with cross-mapping V_{cg} indicates a form of an indirect relationship, V_{dis} still captures unique states of V_{cg} . This high cross-mapping skill speaks to the relevance for including V_{dis} in dynamic data-driven models. Cathode-to-ground voltage V_{cg} initially appears to be driven by cathode current I_{cath} , as might be expected; however, it continues to lose relevant dynamic information of I_{cath} with higher chamber pressures. This could indicate changes in the current paths near the cathode plume, driven by changes in chamber pressure.

B. 6 kW operating condition

The thruster was operated at $6\ \text{kW}$ to explore how the dynamic relationships of the same circuit telemetry are affected under higher discharge-current operating conditions. Specifically, we apply the same causality analysis to the telemetry signals across scaled pressure environments for the thruster operated at $6\ \text{kW}$, which exhibits a peak power spectral density more than two orders of magnitude larger than the peak breathing-mode power spectral density at the $4.5\ \text{kW}$ operation with that peak frequency at $14.52\ \text{kHz}$ for all three signals, as shown in the power spectral density plots of Figs. 6(a)–8(a). Although the breathing-mode oscillation amplitudes are reduced and appear to broaden with increasing pressure, the breathing-mode oscillations remain stable across the three background pressures of 8.5 , 16 , and $25\ \mu\text{Torr}$.

1. Parametric sweep

As a result of the strong breathing-mode oscillation present in all telemetry signals when the thruster is operated at $6\ \text{kW}$, a

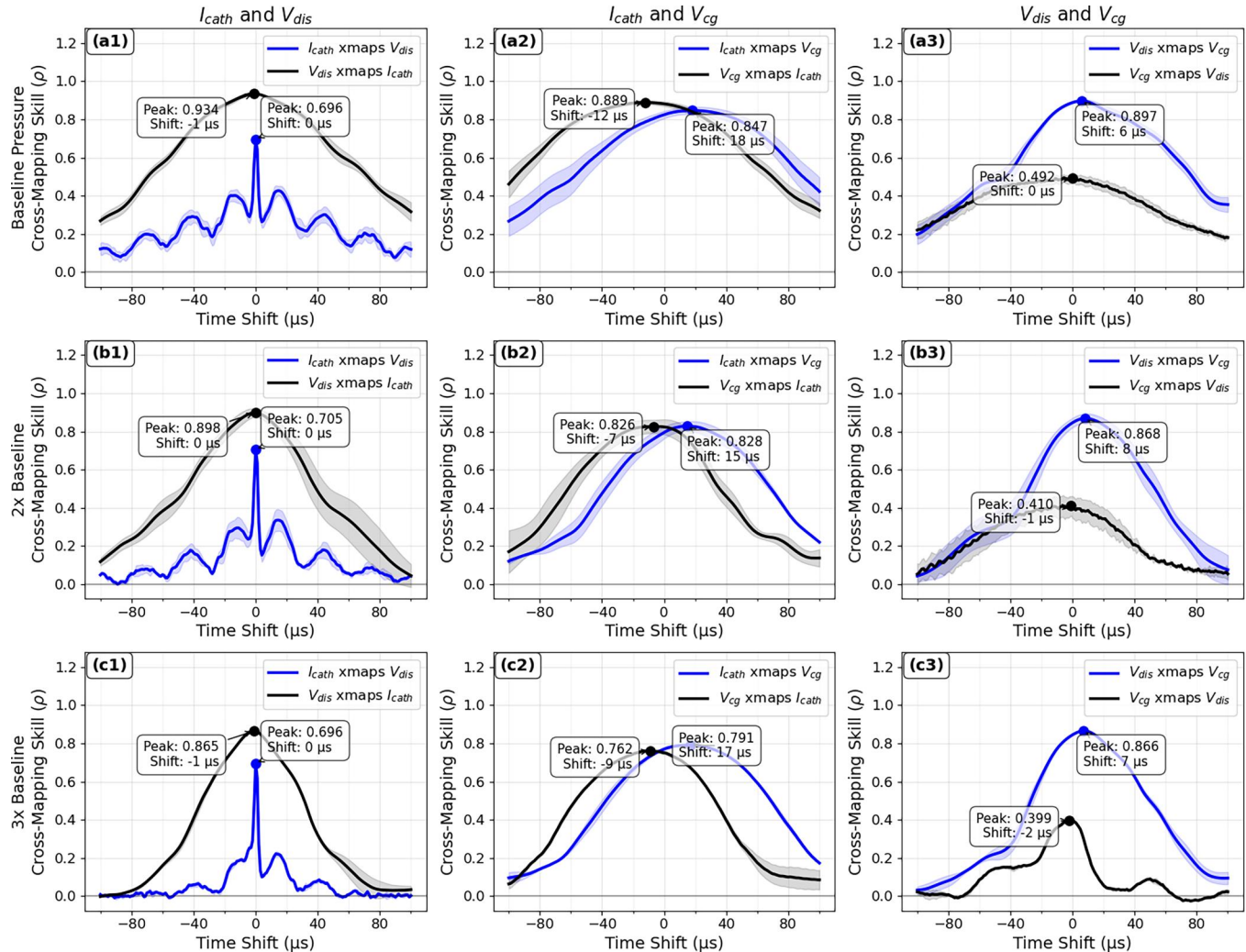


FIG. 5. Extended convergent cross mapping results of telemetry collected from an H9 thruster operated at 4.5 kW and at three background pressure conditions: baseline (a1)–(a3), 2× baseline (b1)–(b3), and 3× baseline (c1)–(c3).

general baseline of high cross-mapping skill across many embedding parameters is observed, driven by the common 14.52 kHz oscillation that dominates the power spectra in each time series. As a result, the information about changes in cross-mapping skills should be evaluated on a relative basis, and therefore so should the dependence on embedding parameters. To aid visualization of relative trends in the effect of embedding parameters on cross-mapping skill at each chamber pressure, an arbitrary minimum cross-mapping skill of 0.8 was applied to the embedding parameter sweeps in Figs. 6–8.

Despite the significant increase in cross-mapping skill among all signals in the 6 kW condition operated at 8.5 μTorr , the embedding parameter regions that yielded the highest cross-mapping skill in Figs. 6(b3)–6(b6) remain similar to those of 4.5 kW shown in Figs. 2(b3)–2(b6) with slight differences in specific peak

embedding parameters. The smaller τ values for I_{cath} and V_{dis} reconstructions in Figs. 6(b1) and 6(b2) are similar, with a preference for larger dimensions for cross-mapping performed by V_{dis} . The regions of high cross-mapping skill between I_{cath} and V_{cg} in Figs. 6(b3) and 6(b4) appear to follow a similar $m\tau$ product relationship, as seen with the 4.5 kW condition.

The background pressure in the chamber was increased to 16 μTorr , roughly twice the base operating pressure at 6 kW, and telemetry was collected once the thruster reached steady-state operation. The resulting embedding parameter sweep, shown in Fig. 7, indicates a reduction in the size of the regions producing the highest cross-mapping skill, similar to the effect observed in the 4.5 kW operation. This trend continues, as shown in Fig. 8, as the background pressure was further increased to 25 μTorr , roughly three times the base operating pressure during 6 kW

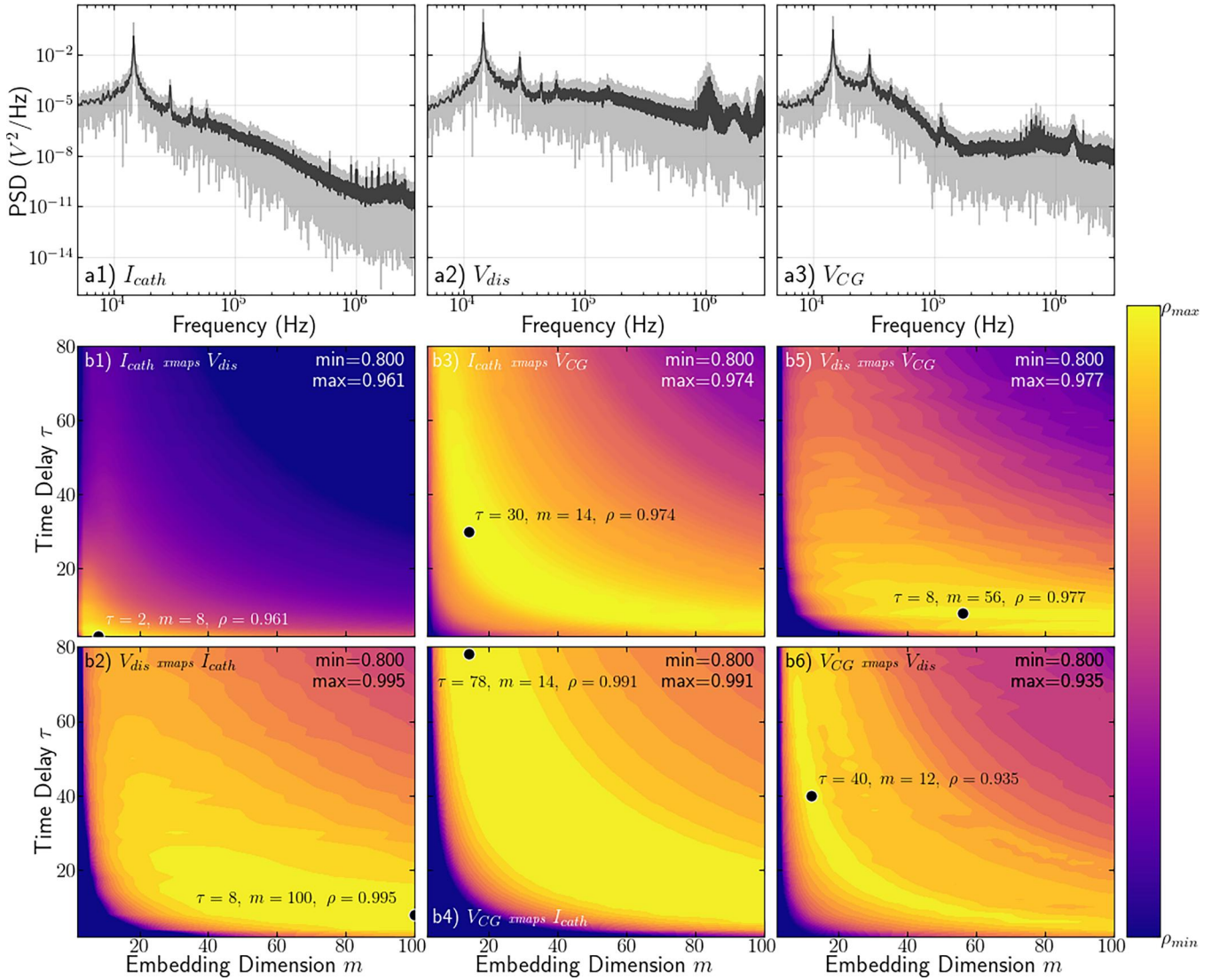


FIG. 6. Power spectral density plots [(a1)–(a3)] and embedding parameter sweep [(b1)–(b6)] of I_{cath} , V_{dis} , and V_{cg} waveforms captured from an H9 thruster operated at 6 kW at baseline pressure, i.e., chamber pressure of $8.5 \mu\text{Torr}$.

thruster operation. There is also a decrease in the peak cross-mapping skill achieved by each of the sweeps with the increase in pressure, which is discussed further in Sec. IV C.

2. Time shift identification

The results of the eCCM analysis applied to the H9 telemetry collected while the thruster was operated at 6 kW are shown in Fig. 9. The resulting cross-mapping skill at each time shift for a given pressure is shown for pressures expressed as multiples of the lowest achievable pressure, referred to as baseline, i.e., $8.5 \mu\text{Torr}$ [(a1)–(a3)], $2\times$ baseline, i.e., $16 \mu\text{Torr}$ [(b1)–(b3)], and $3\times$ baseline,

i.e., $25 \mu\text{Torr}$ [(c1)–(c3)]. Similar to the results from telemetry collected with the thruster operating at 4.5 kW, the discharge voltage appears to reconstruct all other signals best, but continues to show a decrease in peak cross-mapping skill with increasing pressure. Again V_{dis} appears to respond nearly instantaneously to changes in cathode current. The causal relationship between I_{cath} driving V_{cg} in Figs. 9(a2), 9(b2), and 9(c2) appears to be similar in the direction and time shift values to those at the 4.5 kW baseline condition in Fig. 5(a2), but the cathode-to-ground voltage V_{cg} is able to maintain a higher cross-mapping skill than I_{cath} through all increases in pressure. It could be that due to more coherent and larger amplitude breathing-mode oscillations V_{cg} remains causally

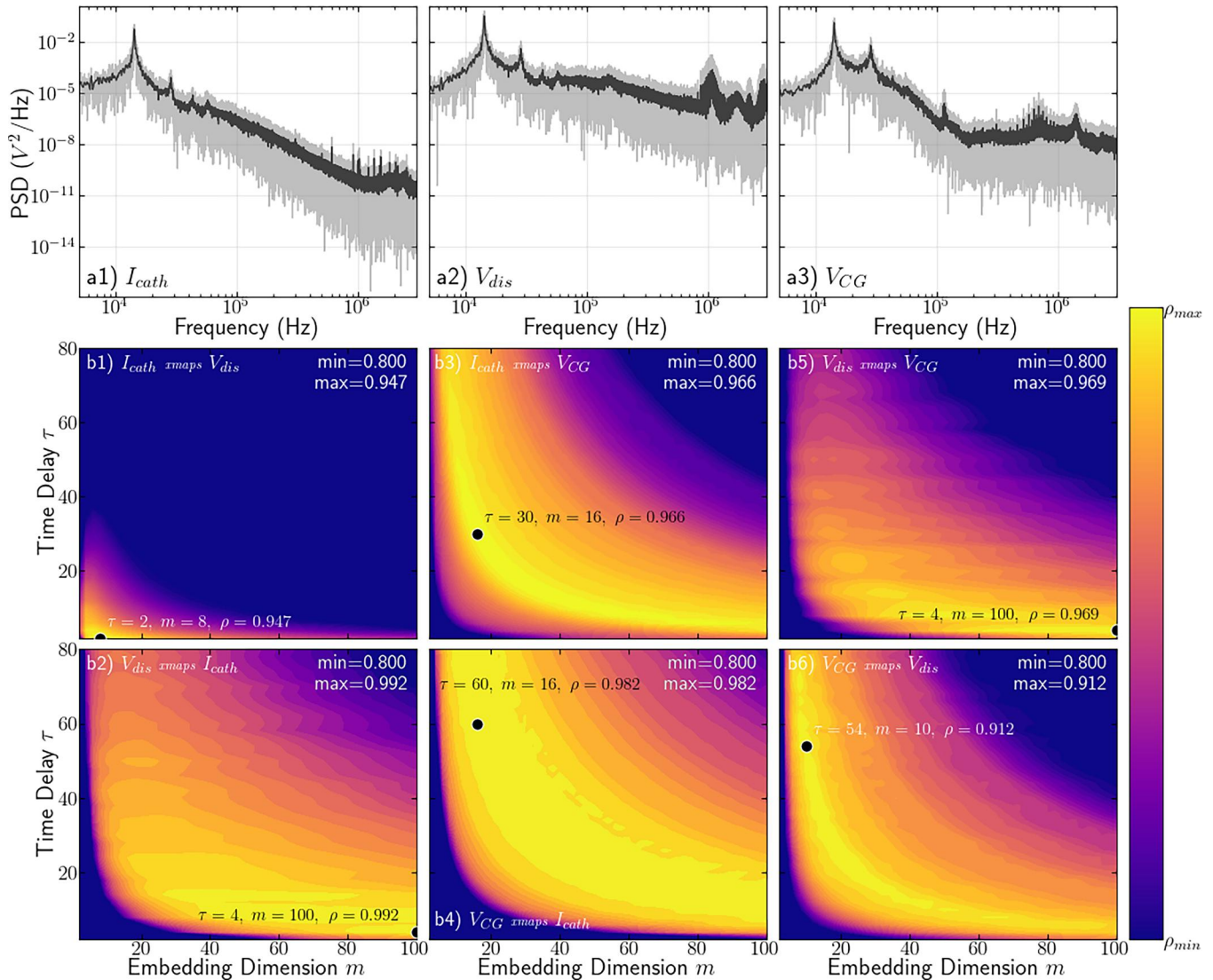


FIG. 7. Power spectral density plots [(a1)–(a3)] and embedding parameter sweep [(b1)–(b6)] of I_{cath} , V_{dis} , and V_{cg} waveforms captured from an H9 thruster operated at 6 kW at twice the baseline pressure, i.e., a chamber pressure of $16 \mu\text{Torr}$.

driven by the primary dynamics in I_{cath} . The results of cross-mapping between V_{dis} and V_{cg} appear to indicate again an indirect relationship, with the peak cross-mapping skill resulting from V_{dis} cross-mapping V_{cg} at a positive time shift. This indicates that the two signals may share common dynamics, but that cathode-to-ground voltage V_{cg} does not capture all the dynamics of the discharge voltage V_{dis} .

C. Effect of background pressure on causality

Understanding the causal relations in HET telemetry can help better select which data to include in data-driven models, but understanding how environmental factors, such as background

chamber pressure, affect these relationships is crucial. If a given signal is selected for its causal relationships with many dynamic data, but those relationships are sensitive to environmental factors such as background pressure, then care must be taken in using the data, as it may no longer contain the relevant information needed to improve state estimation. To this end, we explore how the dynamic relationships among thruster telemetry signals are altered by pressure across the two thruster operating conditions, as shown in Figs. 5 and 9. During thruster operation at 4.5 kW, the peak cross-mapping skill monotonically decreased with increasing chamber pressure for nearly all signals. This trend is illustrated directly in Fig. 10, which plots the maximum cross-mapping skill

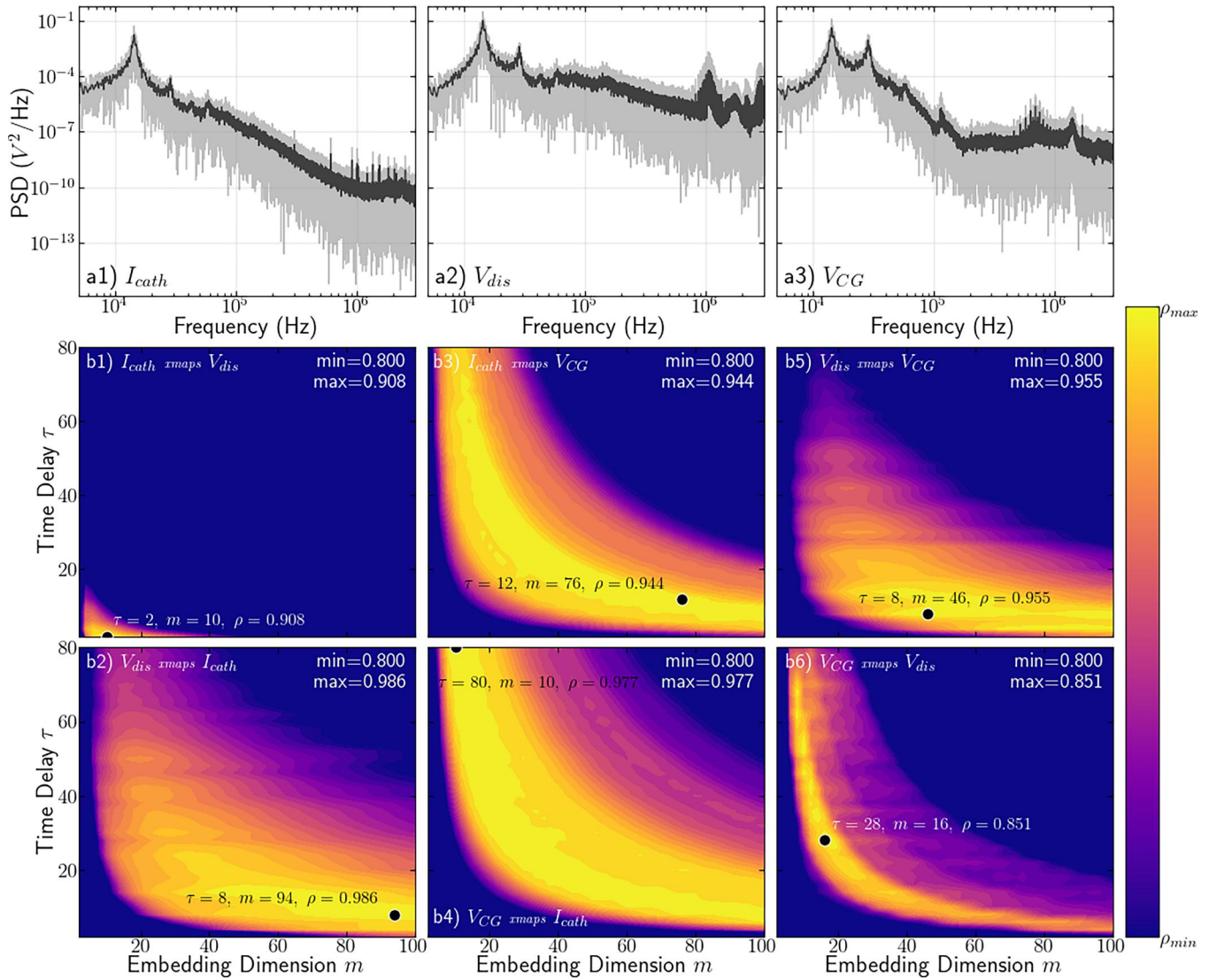


FIG. 8. Power spectral density plots [(a1)–(a3)] and embedding parameter sweep [(b1)–(b6)] of I_{cath} , V_{dis} , and V_{cg} waveforms captured from an H9 thruster operated at 6 kW at three times the baseline pressure, i.e., a chamber pressure of $25 \mu\text{Torr}$.

against background pressure for all pressure conditions. Asymmetric changes in the effect on signal pairs are present in all three, but are most clear in the largest cross-mapping skill change with pressure observed for V_{cg} cross-mapping I_{cath} , in Fig. 10(b). Here, the change in peak cross-mapping is significant enough to drop the reconstruction quality below that of the I_{cath} cross-mapping V_{cg} , indicating that there may not be a clear unidirectional causal relationship between the two, especially at higher pressures. This may be due to changes in the coupling of cathode current to the near-field plasma region, which are influenced by additional charge-exchange current paths to the thruster plume and the chamber wall. If the dynamics of I_{cath} are altered by

elevated charge-exchange ions, the time history of cathode current may not be captured well by the cathode-to-ground voltage. If this effect of chamber pressure is due to the coupling of cathode current to the vacuum chamber wall, it also underscores the importance of the grounded chamber surfaces near the cathode and thruster plume in shaping the relationship between I_{cath} and V_{cg} . Therefore, if data-driven models rely on the causal relationship of V_{cg} for ground testing, it is important to remember that dynamic information of cathode current could be altered with increases in chamber pressure. We note that changes in the cross-mapping of V_{dis} and V_{cg} in Fig. 10(c) appear to show a greater response to the first pressure step than to later increases in

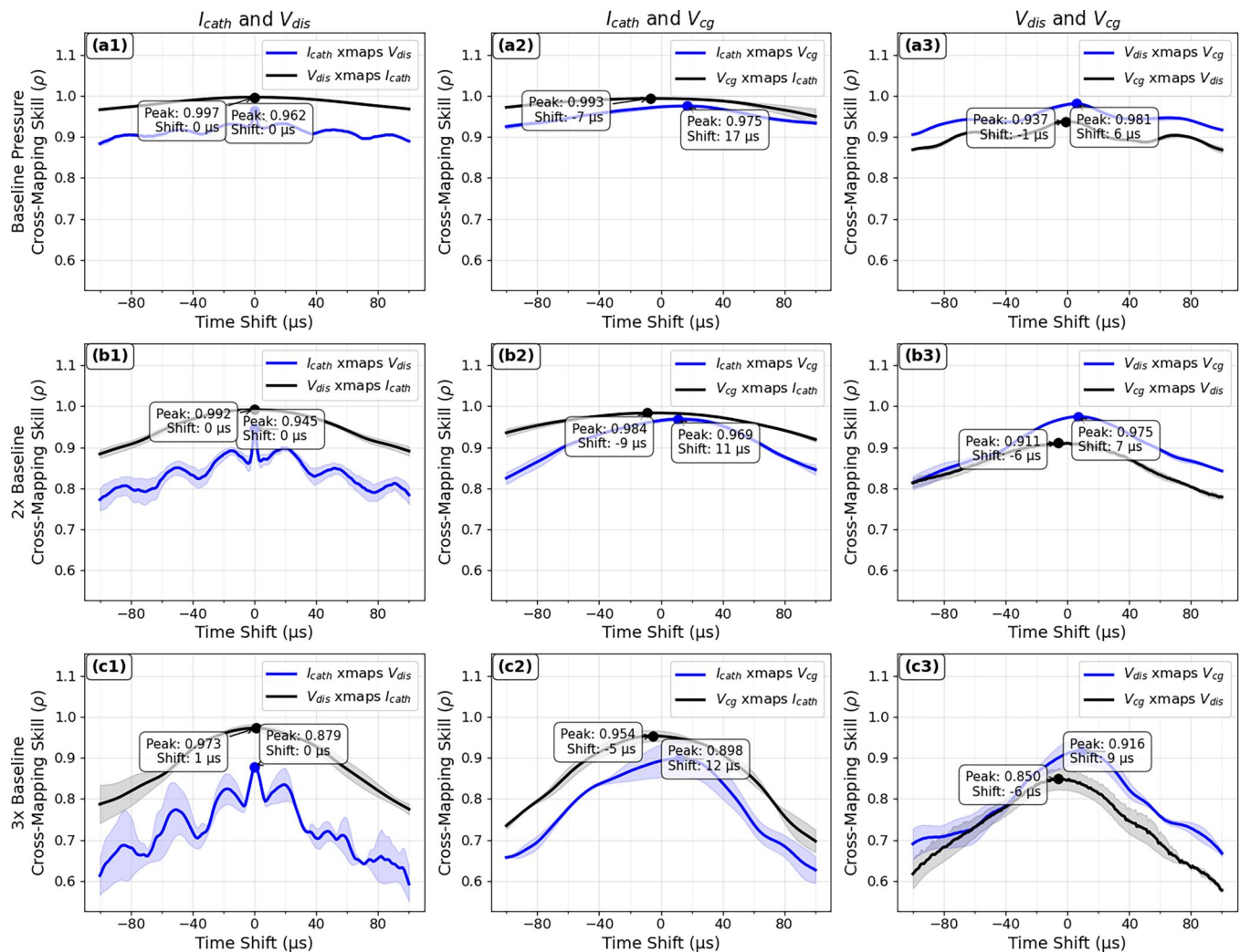


FIG. 9. Extended convergent cross mapping results of telemetry collected from an H9 thruster operated at 6 kW and at three background pressure conditions: baseline (a1)–(a3), 2× baseline (b1)–(b3), and 3× baseline (c1)–(c3).

pressure from the 2×-baseline to the 3×-baseline, indicating that the causal dynamic relationship between the two for the 4.5-kW condition is more sensitive in the first pressure step. This disparity in pressure response indicates a regime change in the dynamic relationship: it is more sensitive at lower pressures but becomes less sensitive once the background pressure is sufficiently high.

The effect of pressure on the peak cross-mapping of telemetry signals collected as the H9 was operated at 6 kW is shown in Fig. 11. As discussed in Sec. IV B, the base cross-mapping skill is significantly higher for this condition due to the dominance of the breathing mode in the system dynamics. As a result, changes in the causal relationships are evaluated at higher levels of relative reconstruction skill. Yet again, a monotonic decline in cross-mapping skill with increasing chamber pressure was observed

across all signals. The largest change in cross-mapping skill is observed between V_{dis} and V_{cg} in Fig. 11(c); however, V_{cg} cross-mappings were strongly influenced by I_{cath} in Fig. 11(b) as well. There appears to be a larger change between the twice baseline (16 μ Torr) and the three times baseline (25 μ Torr) conditions; however, it is unclear whether this change would remain nonlinear with further increases in pressure.

Finally, it is important to note that these signals constitute an incomplete set of the dynamic variables that comprise the dynamic plasma system. As such, the causal relationships identified in eCCM are likely only a subset of the variables comprising the underlying dynamic system. An example of this may be seen in the relationship between V_{dis} and V_{cg} in Figs. 5 and 9. In both operating conditions, V_{dis} can better reconstruct V_{cg} with a

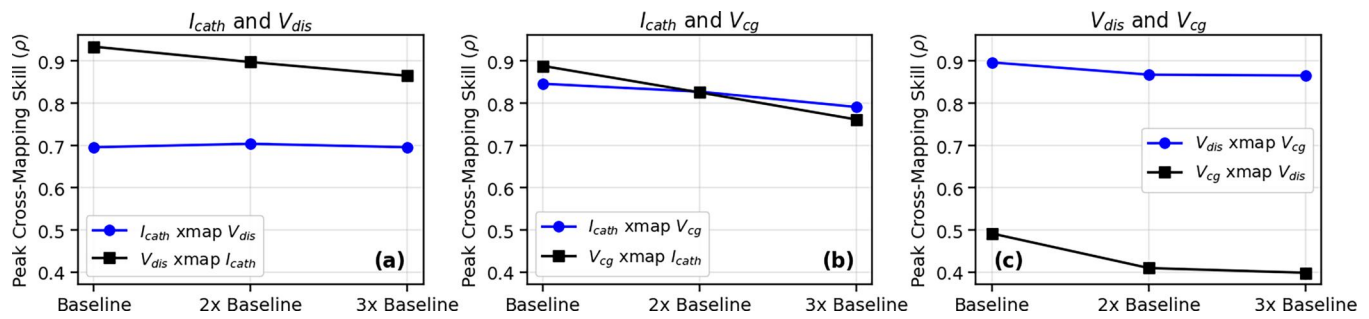


FIG. 10. Maximum cross-mapping skill of each telemetry signal pair, I_{cath} and V_{dis} (a), I_{cath} and V_{cg} (b), as well as V_{cg} and V_{dis} (c) with increasing chamber pressure of an H9 operated at 4.5 kW.

forward time shift, meaning that V_{dis} correctly capture dynamics, although it may also be driven by additional dynamics not shared with V_{cg} , as indicated by the inability of V_{cg} to accurately cross-map V_{dis} to the same degree. In addition, it appears that V_{dis} responds much quicker, $<1 \mu s$ vs $7\text{--}9 \mu s$ to the common dynamics of I_{cath} much more quickly than V_{cg} does, producing results similar to those discussed in the Appendix when two indirect variables respond to a common driver with different response delays. One might propose that a common driver could be I_{cath} ; however, the negative time shifts identified in V_{cg} cross-mapping I_{cath} exceed the delays between V_{dis} and V_{cg} . The apparent indirect relationship between V_{cg} and I_{cath} in Fig. 5(c2) indicates a more complicated dynamic network. A more complete set of dynamic measurements would be needed to further clarify the relationship of the common driver to each signal.

These results were obtained with the thruster-body grounded. In a floating configuration, V_{cg} signals may include stronger baseline coupling to I_{cath} . However, the mechanism driving the pressure-dependent degradation of the dynamic relationship of I_{cath} and V_{cg} —additional facility-coupled current paths opened by elevated background neutrals—remains present in a floated configuration. We, therefore, expect the qualitative trend, namely, a monotonic decrease in peak cross-mapping skill with increasing pressure, to be preserved with a floated body, though the

magnitude of the change in cross-mapping skill (ρ -values) and possibly the time shift of the peaks will differ.

The reductions in ρ between baseline and 3 $\times$ baseline pressure are largest at 4.5 kW—approximately 15% for pairs involving V_{cg} —and smaller but still monotonic at 6 kW, where typical fractional changes fall in the 6%–8% range. These magnitudes should be interpreted against a baseline that is itself influenced by the dominant breathing-mode oscillation at 6 kW: because that oscillation is present in every telemetry channel, it raises ρ for all pairs through generalized synchrony,^{17,40} which, in turn, compresses the absolute $\Delta\rho$ associated with a given physical weakening of the pair-specific causal coupling. To our knowledge, no threshold value of $\Delta\rho$ has been established in the literature at which a change in cross-mapping skill can be said to correspond to a significant alteration in the underlying dynamics, and the present results are offered, in part, as an initial dataset against which such thresholds may eventually be calibrated. The central observation—that ρ decreases monotonically with increasing background pressure across every signal pair and at both power levels—is consistent with the 0.5–0.9 range of ρ values reported in the broader CCM and eCCM literature,^{17,40,49,50} where modest but monotonic changes of this kind have previously been interpreted as evidence of a weakening causal relationship.

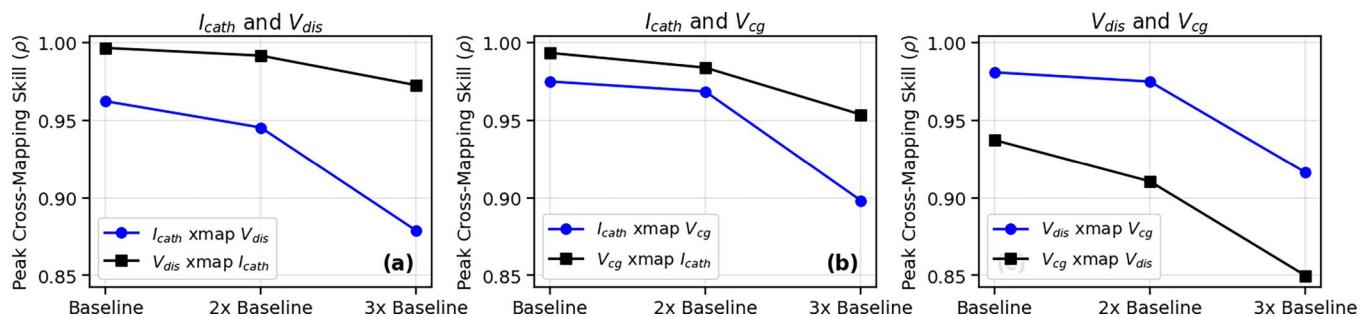


FIG. 11. Maximum cross-mapping skill of each telemetry signal pair, I_{cath} and V_{dis} (a), I_{cath} and V_{cg} (b), as well as V_{cg} and V_{dis} (c) with increasing chamber pressure of an H9 operated at 6.0 kW.

D. Discussion

As shown in the analysis of both the 4.5 and 6 kW operation of the thruster, there is a reduction in the peak cross-mapping skill of the telemetry signals with elevated chamber pressure. It can be hypothesized that this degradation in causality stems from pressure-dependent changes in the current pathways for the cathode. Under normal operation of the thruster in an on-orbit environment, circuit continuity would ensure that cathode current and voltage would remain linked to the anode behavior. However, the presence of background pressure could result in a higher near-field plasma density that enables current flow of electrons to the electrically conducting vacuum facility walls or different components, e.g., thruster body. In this scenario, discharge ions can be neutralized by electrons on facility surfaces with cathode-supplied electrons arriving at the facility elsewhere, partially decoupling the discharge behavior from the cathode current. To better understand if this is a reasonable proposal for the observed behavior, a larger set of system variables would need to be included in the analysis.

Expanding the set of available signals to better capture the dynamical network could include both high-speed probes and optical measurements coupled with expanded telemetry to improve our ability to capture changes in the underlying dynamics. This would help provide a clearer understanding of how the environmental factors, such as chamber pressure, alter the dynamic relationships that exist between the discharge, cathode, and near-field plasmas. Another point of interest is to explore how to incorporate environmental conditions that affect causal relations into multi-modal data-driven frameworks that allow the use of signals whose dynamic relationships are sensitive to such factors (e.g., gain adjustments to Kalman filters under specific environmental conditions).

The ability to tie direct changes in dynamic relationships of telemetry to the underlying physics of the thruster operation is limited due to the lack of time-resolved plasma property measurements. Future measurements will need to focus on establishing these links between telemetry and plasma property measurements to advance the physical understanding of how changes in the underlying physics manifest as changes in the dynamic causality. For now, if we adopt the working assumption that the cathode-to-ground voltage V_{cg} is indicative of downstream plume dynamics, while the discharge voltage V_{dis} is representative of near-field discharge dynamics, then the larger pressure-induced decoupling observed between V_{dis} and V_{cg} at 4.5 kW, relative to 6 kW, can be interpreted as a more significant separation of near-field and far-field dynamics at the lower power level. The comparatively smaller decoupling observed at 6 kW is consistent with the stronger breathing-mode oscillation under that condition, which requires coherent periodic changes in both the near-field discharge and the downstream plume, thereby maintaining tighter dynamic coupling between the two regions even as the background pressure is increased.

It is important to clarify what the present framework does, and does not, enable with respect to the long-standing problem of extrapolating ground-test results to on-orbit pressure environments. The eCCM analysis reported here does not itself constitute an extrapolation tool nor can the pressure trends in ρ be projected

to pressures below the baseline condition of the vacuum facility. What the framework does provide is a quantitative, signal-level description of which causal relationships among electric circuit components are sensitive to background pressure. This information is directly useful for the development of dynamic, physics-based models of HET operation: it identifies the signal pairs whose coupling must be represented accurately in any reduced-order or state-space model intended to capture facility-effect sensitivity (e.g., boundary conditions), and it flags those couplings for which environmental factors other than pressure may dominate.

V. CONCLUSION

Extended convergent cross mapping was applied to three externally accessible telemetry signals, cathode current I_{cath} ; discharge voltage V_{dis} ; and cathode-to-ground voltage V_{cg} , collected from an H9 magnetically shielded Hall-effect thruster operated on krypton in VTF-2 at two power levels and three background pressures. Across nearly every signal pair and both operating conditions, the peak cross-mapping skill decreases monotonically with increasing chamber pressure, indicating that facility pressure measurably alters the causal relationships among these signals. Of the three, V_{dis} consistently produces the strongest cross-maps of the other two and is, therefore, the most dynamically informative of the signals analyzed here, although its relationship to V_{cg} appears to be indirect rather than direct.

The reductions in peak cross-mapping skill reported here are modest in absolute magnitude but monotonic across signal pairs, and—particularly at 6 kW, where the breathing-mode contribution elevates the baseline through generalized synchrony—can be interpreted as a deterioration of pair-specific causal information rather than a numerical artifact. We emphasize that the primary contribution of this work is the continued development of the eCCM framework as a tool for characterizing dynamic relationships in Hall-effect thruster telemetry; rigorous connection of the observed causal trends to specific discharge physics will require co-measured dynamic plasma property data and coupling to physics-based models, which we leave to future work. While these results quantify how causal coupling between telemetry signals evolves with chamber pressure over the tested range, they are not intended to provide a direct means of extrapolating thruster dynamics to on-orbit pressure levels. Instead, they are intended to provide an empirical baseline against which future dynamic, physics-based models can be developed, tested, and refined.

The practical implication is that signal selection performed in one facility environment may not preserve the same dynamic information in another: because I_{cath} , V_{dis} , and V_{cg} all exhibit pressure-sensitive causal coupling, data-driven models that rely on these signals should be validated across facilities or across a range of chamber pressures before their causal assumptions are carried over. Where the available measurement set is constrained, as in most flight-representative testing, eCCM provides a quantitative basis for selecting signals whose dynamic relationships are strongest under the conditions of interest, and for flagging those whose coupling is most vulnerable to environmental change. Expansion of the analyzed signal set to include high-speed in-vacuum probes and optical diagnostics, and direct comparison between grounded-

and floated-body configurations are identified as natural follow-on studies.

ACKNOWLEDGMENTS

The work is supported by NASA through the Joint Advanced Propulsion Institute, a NASA Space Technology Research Institute, under Grant No. 80NSSC21K1118.

AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Seth J. Thompson: Conceptualization (equal); Data curation (equal); Formal analysis (equal); Investigation (lead); Methodology (equal); Writing – original draft (lead); Writing – review & editing (equal). **Kentaro Hara:** Conceptualization (equal); Data curation (equal); Formal analysis (equal); Funding acquisition (equal); Investigation (equal); Methodology (equal); Writing – original draft (equal); Writing – review & editing (equal). **Janice D. Cabrera:** Conceptualization (supporting); Data curation (equal); Investigation (supporting). **Mitchell L. R. Walker:** Funding acquisition (equal); Project administration (equal); Supervision (supporting). **John D. Williams:** Conceptualization (supporting); Funding acquisition (equal); Project administration (equal); Supervision (equal).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

APPENDIX: EXAMPLE WITH GENERAL SYNCHRONY

To evaluate our eCCM analysis, we use an example problem in which a delayed response between variables can be controlled to assess proper identification via the time shift, l , in the eCCM process. We use a set of equations that contain two variables X and Y , driven by a third causal variable Z with a delay k in the

response of X to Z ,

$$\begin{aligned} Z(t+1) &= Z(t)(A_z - A_z Z(t)), \\ Y(t+1) &= Y(t)(A_y - A_y Y(t) - B_{yz} Z(t)), \\ X(t+1) &= X(t)(A_x - A_x X(t) - B_{xz} Z(t-k)). \end{aligned} \quad (\text{A1})$$

To generate the time series data used for evaluating our eCCM framework, all coefficients are kept the same, $A_z = 3.8$, $A_y = 3.1$, $A_x = 3.1$, $B_{yz} = 0.9$, $B_{xz} = 0.5$, and the initial values of $X(0)$, $Y(0)$, $Z(0) = 0.3$. The cross-mapping and time shift plots of all variables correctly identified a causal relationship with appropriate directional shifts for Y and X both influenced by Z . However, this process highlighted a key result. In a case where two variables, such as X and Y , are strongly driven by a third common variable Z with differing delay in response (when $k = 2$), an apparent causal relationship between the indirect variables can be observed, with a time shift corresponding to the response disparity between the two variables to the common driver. This is especially relevant to systems where it is possible that not all system variables would be evaluated in the causal network, such as in our use of a limited subset of telemetry signals. In the example problem where all variables are reconstructed by one another, the reconstructions are still lower in quality than those of the correct causal directions $Z \rightarrow X$ and $Z \rightarrow Y$. However, when the direct relationships among all system variables cannot be evaluated, ambiguity may arise in the causal relationships inferred solely from time-shift data and reconstruction quality. As a result, we must understand that apparent causal (unidirectional or bidirectional) relationships with appropriate time delays may also arise from indirect relationships between variables that share a common driving variable with differing response delays.

For our analysis of the time series generated from Eq. (A1), we also conducted a parametric sweep to identify optimal embedding parameters for each signal set. Interestingly, the optimal value τ was 2 in all reconstructions, while the embedding dimension varied depending on the signals being reconstructed: the optimal embedding dimension was 4 for reconstructions in the correct causal direction and 5–6 (occasionally 7 when $k = 2$) in the opposite direction. This increase in dimension can broaden

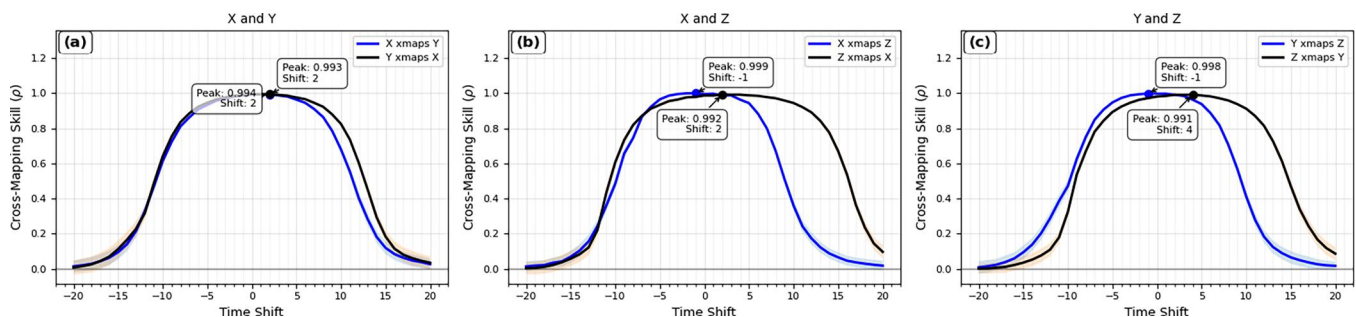


FIG. 12. Extended cross-mapping results of Eq. (A1) with the delay $k = 0$, with ambiguous results of how X and Y relate to each other (a), but with accurate identification of Z 's causal effect on X and Y [(b) and (c)].

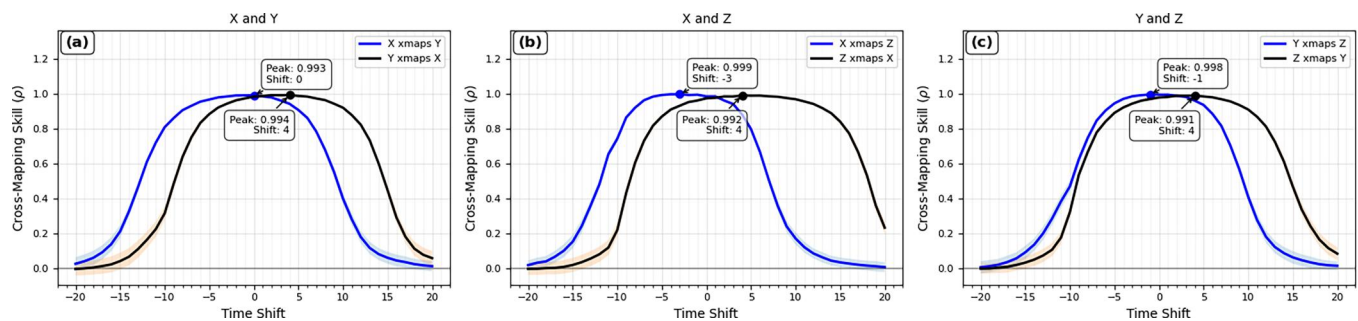


FIG. 13. Extended cross-mapping results of Eq. (A1) with the delay $k=2$ used to demonstrate how information delays can complicate the interpretation of uncoupled variables X and Y (a), while showing disparity in the time shift between X and Z (b) and Y and Z (c), better detailing the relationship between the common driving variable.

out the width of the higher cross-mapping skill vs time shift, as seen in Figs. 12 and 13; however, the correct delays were still identified.

The first application of the eCCM on the time series generated from Eq. (A1) with no response delay ($k=0$) is shown in Fig. 12. Plots (b) and (c) show the correct identification of the causal direction, with the highest reconstruction quality occurring for X and Y cross-mapping Z , with a negative shift. This indicated a strong unidirectional causal relationship. The time shift in the forward direction appears to slightly over predict, which may be due to the increased dimensionality of the embeddings used to reconstruct signals in the non-causal direction, as well as the centering of the embedding as discussed in Ref. 17. The reconstructions of X and Y cross-mapping one another are both forward-shifted, indicating an inconclusive result, as may be expected for indirect variables driven by a strong common driver.

The response delay of X to Z in Eq. (A1) was changed to $k=2$, and all three signals were cross-mapped again, with the optimum embedding dimension found for each signal being cross-mapped, shown in Fig. 13. The correct identification of the time delay and causal direction for X and Y cross-mapping Z are shown in plots (b) and (c). However, unlike the results with $k=0$, when $k=2$, the result of X cross-mapping Y appears to indicate a causal relationship with no time shift. Were it not for the tightly coupled unidirectional relationship in plots (b) and (c), one might incorrectly attribute the relationship $Y \rightarrow X$, due to the disparity between the two variables' responses to Z , between the two variables. This result should be kept top of mind when an incomplete set of system variables is examined: causal relationships identified in a subset of system signals may result from indirectly related variables that share a common driver with a response disparity.

REFERENCES

- ¹D. Y. Oh *et al.*, "Design and development of the Psyche Spacecraft for NASA's Discovery Program," *Space Sci. Rev.* **221**(7), 96 (2025).
- ²I. D. Kaganovich *et al.*, "Physics of $E \times B$ discharges relevant to plasma propulsion and similar technologies," *Phys. Plasmas* **27**(12), 120601 (2020).
- ³K. Diamant, R. Spector, E. Beiting, J. Young, and T. Curtiss, "The effects of background pressure on Hall thruster operation," AIAA Paper No. 2012-3735, 2012.
- ⁴K. D. Diamant, R. Liang, and R. L. Corey, "The effect of background pressure on SPT-100 Hall thruster performance," AIAA Paper No. 2014-3710, 2014.
- ⁵J. S. Snyder, G. Lenguito, J. D. Frieman, T. W. Haag, and J. A. Mackey, "Effects of background pressure on SPT-140 Hall thruster performance," *J. Propuls. Power* **36**(5), 668–676 (2020).
- ⁶W. Huang, H. Kamhawi, and P. Y. Peterson, "Effects of background pressure and electrical configuration on the velocity field of the HERMeS Hall thruster," AIAA Paper No. 2020-3616, 2020.
- ⁷J. A. Walker, S. J. Langendorf, M. L. R. Walker, V. Khayms, D. King, and P. Peterson, "Electrical facility effects on Hall current thrusters: Electron termination pathway manipulation," *J. Propuls. Power* **32**(6), 1365–1377 (2016).
- ⁸J. Walker, D. Lev, M. L. R. Walker, V. Khayms, and D. King, "Electrical characteristics of a Hall effect thruster body in a vacuum facility testing environment," *J. Electr. Propuls.* **1**(1), 18 (2022).
- ⁹D. R. Jovel, "Impedance characterization of a Hall effect thruster discharge in a ground-based vacuum test facility," Ph.D. dissertation (Georgia Institute of Technology, 2024).
- ¹⁰L. Brieda, "Molecular outgassing and deposition in EP applications," in *34th International Electric Propulsion Conference*, IEPC-2015-361 (Electric Rocket Propulsion Society, 2015).
- ¹¹M. Choi, J. T. Yim, G. J. Williams, D. A. Herman, and J. H. Gilland, "Hybrid-particle-in-cell simulation of backspattered carbon transport in the near-field plume of a Hall thruster," in *35th International Electric Propulsion Conference*, IEPC-2017-537 (Electric Rocket Propulsion Society, 2017).
- ¹²R. B. Lobb, J. E. Polk, R. R. Hofer, V. H. Chaplin, and B. Jorns, "Accelerating 23,000 h of ground test backspattered carbon on a magnetically shielded Hall thruster," AIAA Paper No. 2019-3989, 2019.
- ¹³E. T. Dale and B. A. Jorns, "Non-invasive time-resolved measurements of anomalous collision frequency in a Hall thruster," *Phys. Plasmas* **26**(1), 013516 (2019).
- ¹⁴D. E. Troyetsky, C. M. Greve, S. Tsikata, and K. Hara, "State estimation of the dynamic behavior of plasma properties in a Hall effect thruster discharge," *J. Phys. D: Appl. Phys.* **56**(44), 444001 (2023).
- ¹⁵C. M. Greve, K. Hara, R. S. Martin, D. Q. Eckhardt, and J. W. Koo, "A data-driven approach to model calibration for nonlinear dynamical systems," *J. Appl. Phys.* **125**(24), 244901 (2019).
- ¹⁶T. Zivri and K. Hara, "Estimation of in-space thruster performance of a Hall effect thruster through extrapolation of plasma states using extended Kalman filter," *J. Appl. Phys.* **138**(16), 163305 (2025).
- ¹⁷H. Ye, E. R. Deyle, L. J. Gilarranz, and G. Sugihara, "Distinguishing time-delayed causal interactions using convergent cross mapping," *Sci. Rep.* **5**(1), 14750 (2015).

- ¹⁸C. M. Greve and K. Hara, "Estimation of plasma properties using an extended Kalman filter with plasma global models," *J. Phys. D: Appl. Phys.* **55**(25), 255201 (2022).
- ¹⁹E. Choueiri, "Plasma oscillations in Hall thrusters," *Phys. Plasmas* **8**, 1411–1426 (2001).
- ²⁰K. Hara, M. J. Sekerak, I. D. Boyd, and A. D. Gallimore, "Perturbation analysis of ionization oscillations in Hall effect thrusters," *Phys. Plasmas* **21**(12), 122103 (2014).
- ²¹E. T. Dale and B. A. Jorns, "Experimental characterization of Hall thruster breathing mode dynamics," *J. Appl. Phys.* **130**(13), 133302 (2021).
- ²²T. Lafleur, P. Chabert, and A. Bourdon, "The origin of the breathing mode in Hall thrusters and its stabilization," *J. Appl. Phys.* **130**(5), 053305 (2021).
- ²³K. Hara and S. Tsikata, "Cross-field electron diffusion due to the coupling of drift-driven microinstabilities," *Phys. Rev. E* **102**(2), 23202 (2020).
- ²⁴J. P. Boeuf and L. Garrigues, " $E \times B$ electron drift instability in Hall thrusters: Particle-in-cell simulations vs. theory," *Phys. Plasmas* **25**(6), 061204 (2018).
- ²⁵B. A. Jorns, I. G. Mikellides, and D. M. Goebel, "Ion acoustic turbulence in a 100-A LaB6 hollow cathode," *Phys. Rev. E* **90**(6), 63106 (2014).
- ²⁶K. Hara and C. Treece, "Ion kinetics and nonlinear saturation of current-driven instabilities relevant to hollow cathode plasmas," *Plasma Sources Sci. Technol.* **28**(5), 055013 (2019).
- ²⁷S. Tsikata, K. Hara, and S. Mazouffre, "Characterization of hollow cathode plasma turbulence using coherent Thomson scattering," *J. Appl. Phys.* **130**(24), 243304 (2021).
- ²⁸A. P. Scott and D. M. Goebel, "Hollow cathode discharge instability onset in electric thrusters," *J. Appl. Phys.* **135**(12), 123301 (2024).
- ²⁹C. M. Greve and C. Marsh, "Evaluating Hall thruster startup transients using dynamics-based embeddings," *J. Electr. Propuls.* **4**(1), 36 (2025).
- ³⁰W. Hargus and B. Pote, "Examination of a Hall thruster start transient," AIAA Paper No. 2002-3956, 2002.
- ³¹M. G. Allen, P. J. Roberts, and B. A. Jorns, "Direct measurements of the pressure dependence of the electron temperature in a magnetically shielded Hall thruster," in *38th International Electric Propulsion Conference*, IEPC-2024-821 (Electric Rocket Propulsion Society, 2024).
- ³²J. Cabrera, D. Jovel, and M. L. R. Walker, "Pressure facility effects on the impedance characteristics of a 9-kW HET," in *38th International Electric Propulsion Conference*, IEPC-2024-808 (Electric Rocket Propulsion Society, 2024).
- ³³R. R. Hofer, S. E. Cusson, R. B. Lobb, and A. D. Gallimore, "The H9 magnetically shielded Hall thruster," in *35th International Electric Propulsion Conference*, IEPC-2017-232 (Electric Rocket Propulsion Society, 2017).
- ³⁴P. Y. Peterson *et al.*, "NASA's HERMeS Hall thruster electrical configuration characterization," AIAA Paper No. 2016-5027, 2016.
- ³⁵R. R. Hofer and J. R. Anderson, "Finite pressure effects in magnetically shielded Hall thrusters," AIAA Paper No. 2014-3709, 2014.
- ³⁶J. W. Dankanich, M. Walker, M. W. Swiatek, and J. T. Yim, "Recommended practice for pressure measurement and calculation of effective pumping speed in electric propulsion testing," *J. Propuls. Power* **33**(3), 668–680 (2017).
- ³⁷C. M. Greve, M. Majji, and K. Hara, "Real-time state estimation of low-frequency plasma oscillations in Hall effect thrusters," *Phys. Plasmas* **28**(9), 093509 (2021).
- ³⁸C. W. J. Granger, "Investigating causal relations by econometric models and cross-spectral methods," *Econometrica* **37**(3), 424–438 (1969).
- ³⁹F. Takens, "Detecting strange attractors in turbulence," in *Dynamical Systems and Turbulence, Warwick 1980*, edited by D. Rand and L.-S. Young (Springer, Berlin, 1981), pp. 366–381.
- ⁴⁰G. Sugihara *et al.*, "Detecting causality in complex ecosystems," *Science* **338**(6106), 496–500 (2012).
- ⁴¹G. Sugihara and R. M. May, "Nonlinear forecasting as a way of distinguishing chaos from measurement error in time series," *Nature* **344**(6268), 734–741 (1990).
- ⁴²D. Eckhardt, J. Koo, R. Martin, M. Holmes, and K. Hara, "Spatiotemporal data fusion and manifold reconstruction in Hall thrusters," *Plasma Sources Sci. Technol.* **28**(4), 045005 (2019).
- ⁴³M. Baird, R. McGee-Sinclair, K. Lemmer, and W. Huang, "Time-resolved ion energy measurements using a retarding potential analyzer," *Rev. Sci. Instrum.* **92**(7), 073306 (2021).
- ⁴⁴S. J. Thompson, Z. K. Robertson, S. C. Farnell, C. C. Farnell, C. C. Farnell, and J. D. Williams, "Time-resolved measurements of plasma properties in the beam plasma of a Hall thruster," in *38th International Electric Propulsion Conference*, IEPC-2024-818 (Electric Rocket Propulsion Society, 2024).
- ⁴⁵A. Thomas and K. Lemmer, "Time-resolved ion energy measurements using a retarding potential analyzer for electric propulsion applications," *Rev. Sci. Instrum.* **95**(2), 023505 (2024).
- ⁴⁶P. J. Roberts, V. H. Chaplin, and B. A. Jorns, "Time-resolved laser diagnostics in Hall thrusters with time-delay embedding data fusion," in *38th International Electric Propulsion Conference*, IEPC-2024-596 (Electric Rocket Propulsion Society, 2024).
- ⁴⁷P. A. P. Moran, "The statistical analysis of the Canadian lynx cycle," *Aust. J. Zool.* **1**(3), 291–298 (1953).
- ⁴⁸C. E. Huerta, R. S. Martin, D. Q. Eckhardt, and J. W. Koo, "Determining causality in Hall effect thrusters using extended convergent cross mapping, part I," *Plasma Sources Sci. Technol.* **30**(7), 075004 (2021).
- ⁴⁹C. E. Huerta, R. S. Martin, D. Q. Eckhardt, and J. W. Koo, "Determining causality in Hall effect thrusters using extended convergent cross mapping: II," *Plasma Sources Sci. Technol.* **31**(3), 035015 (2022).
- ⁵⁰C. Huerta, C. Greve, and A. Wong, "Comparison of causality determination techniques in studying Hall-effect thrusters," *J. Electr. Propuls.* **3**(1), 23 (2024).
- ⁵¹H. Kantz and T. Schreiber, *Nonlinear Time Series Analysis* (Cambridge University Press, 2003).
- ⁵²L. Cao, "Practical method for determining the minimum embedding dimension of a scalar time series," *Phys. D: Nonlinear Phenom.* **110**(1–2), 43–50 (1997).
- ⁵³R. S. Martin, C. M. Greve, C. E. Huerta, A. S. Wong, J. W. Koo, and D. Q. Eckhardt, "A robust time-delay selection criterion applied to convergent cross mapping," *Chaos* **34**(9), 093110 (2024).