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A COMPARISON OF THE METHODS OF DETERMINING ELECTRON DENSITIES IN AFTERGLOW PLASMAS FROM LANGMUIR PROBE CHARACTERISTICS

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Abstract—A diversity of opinion exists at the present time concerning the interpretation of single Langmuir probe characteristics, in particular with respect to the location of space potential and the determination of charged particle number densities from the characteristics. This work describes measurements made with very small Langmuir probes in gaseous afterglow plasmas at room temperature where conditions for a study of this kind are expected to offer particular advantages. A comparison is made of the values of electron densities obtained from the characteristics using six methods which are commonly used. Good agreement is found to exist between those deduced from the orbital limited characteristics in the accelerating region for electrons and those calculated from the probe current at the inflexion point of the characteristics. That these are also in tolerable agreement with those obtained using the intersecting tangents method is thought to be fortuitous.

1. INTRODUCTION

SINCE Langmuir's classic theoretical and experimental work in 1923 (LANGMUIR *et al.*, 1923, 1924; MOTT-SMITH and LANGMUIR, 1926) the electrostatic probe has been widely used as a diagnostic tool both in laboratory plasmas and, more recently, in ionospheric studies. Its main role has been that of measuring such plasma parameters as the number density, temperature (LOEB, 1960) and energy distributions (SLOANE and MACGREGOR, 1934; MEDICUS, 1956; LECKEY *et al.*, 1963; BOYD and TWIDDY, 1959) of the charged particles comprising the plasma, though it has also been used to determine reflection coefficients (LAMAR and COMPTON, 1931) and electronic work functions of probe materials (VAN VOORHIS, 1927; VAN VOORHIS and COMPTON, 1930). Although there is good qualitative agreement between the theoretically predicted current–voltage characteristics of the probe and those obtained in practice, differences do occur which have led to criticisms of the use of probes (LOEB, 1960). The most serious of these is that the current drawn by the probe disturbs the plasma. In consequence there exists a diversity of opinion concerning the interpretation of probe characteristics, in particular with respect to the values of space potential and charged particle number density deduced from the characteristics. Single Langmuir probes are, at present, being used in this laboratory to study diffusion and dissociative recombination processes occurring in afterglow plasmas, a situation where conditions for the reliable use of probes are considered by the authors to offer particular advantages (see Section 4). Since probe studies of dissociative recombination processes require an absolute determination of the undisturbed electron density, observations were made to ascertain the degree of consistency between the several methods which have been used to obtain electron density values from probe characteristics. Although the double probe is normally recommended for use in decaying plasmas in preference to the single probe on account of the decreased current drain from the plasma (JOHNSON and MALTER, 1950), the authors consider that the depletion problems associated with the very small single probe used in the present experiments are not important.

2. THEORY OF THE CYLINDRICAL LANGMUIR PROBE

As Langmuir probe theory is so well documented (LANGMUIR *et al.*, 1923, 1924; MOTT-SMITH and LANGMUIR, 1926) only the relevant equations governing the behaviour of probes in a Maxwellian plasma will be considered in this section. Further, the discussion will be limited solely to the theory of the cylindrical probe since it was a cylindrical probe that was chosen for the present series of experiments (see Section 5).

When the electronic mean free path is greater than the radius of the sheath (a) surrounding the probe, the electron current to a probe biased negatively with respect to space potential is given by

$$i_e = An_e e \sqrt{\left(\frac{kT_e}{2\pi m_e}\right)} \exp\left(\frac{-eV}{kT_e}\right), \tag{1}$$

where A is the collecting area of the probe (m^2)
n_e is the undisturbed electron density in the plasma (m^{-3})
T_e is the electron temperature (°K)
V is the magnitude of the probe potential with respect to space potential (V), and the other symbols have their usual meaning.

It can be seen from equation (1) that at space potential the current to the probe is given by

$$i_e = An_e e \sqrt{\left(\frac{kT_e}{2\pi m_e}\right)} = Aj_e, \tag{2}$$

where j_e is the random electron current density ($A.m^{-2}$).

The electron current to a cylindrical probe biased positively with respect to space potential is described by

$$i_e = Aj_e\left[\frac{a}{r}(1 - \operatorname{erf}[\sqrt{\phi}]) + \exp(\eta) \,.\, \operatorname{erf}(\sqrt{[\phi + \eta]})\right], \tag{3}$$

where

$$\eta = \frac{eV}{kT_e}: \qquad \phi = \frac{r^2\eta}{(a^2 - r^2)}:$$

r is the probe radius and the error function is given by

$$\operatorname{erf} x = \frac{2}{\sqrt{\pi}}\int_x^\infty \exp(-y^2)\, dy.$$

This equation is difficult to manipulate but a graphical representation of it has been published by LANGMUIR and COMPTON (1931). Analytic approximations, however, exist for the error function provided that the lower limit x of the error function integral is either very large or very small.

It can be shown that when $\phi \geqslant 4$, or, correspondingly, when $a/r \sim 1$, equation (3) reduces to

$$i_e = A_s j_e, \tag{4}$$

where A_s is the area of the sheath surrounding the probe.

In this situation the current to the probe is said to be space charge limited and can also be described by the Child–Langmuir equation (LANGMUIR, 1913; LANGMUIR and BLODGETT, 1923).

Under conditions where $0 \leqslant \phi \leqslant 1$ and $\eta \geqslant 4$ or alternatively $a/r \gg 1$, the electron current to the probe is termed orbital limited and is given by

$$i_e^2 = \frac{2A^2 n_e^2 e^2}{\pi^2 m_e} (eV + kT_e). \tag{5}$$

Until recently, no analytical expressions have been formulated for the radius of the space sheath surrounding a cylindrical or spherical probe, thus rendering difficult any check on the conditions implicit in equations (4) and (5). BETTINGER and WALKER (1965), from the computer calculations of WALKER (1965), have now derived such expressions, the expression for the radius of the sheath about a cylindrical probe being

$$a = 1{\cdot}66 h \eta^{3/4} + r, \tag{6}$$

where h is the Debye shielding length (DEBYE and HÜCKEL, 1923; MCDANIEL, 1964) given by

$$h = \left[\frac{\varepsilon \varepsilon_0 k T_e}{e^2 n_e}\right]^{1/2} \text{ m.k.s. units} \tag{7}$$

ε being the dielectric constant of the medium and ε_0 the permittivity of free space.

From a consideration of the equations given in this section, methods are indicated by which values of the electron density and space potential may be determined. The electron temperature can be obtained from the slope of a graph of $\ln i_e$ plotted against probe voltage in the retarding region [equation (1)]. The electron density may be determined from the current flowing to the probe at space potential [equation (2)] or, alternatively under orbital limited conditions, it may be calculated from the slope of a graph of i_e^2 vs. V in the accelerating region [equation (5)].

The above equations are only valid for electron currents to the probe. However, in practice, the positive ion contribution to the total current collected by the probe immersed in a plasma has to be taken into account. Although there exist sophisticated theories describing the behaviour of positive ion collection by a probe (for example VAN ECK and KINDERDIJK, 1967), the ion contribution is normally assumed to be that given by the extrapolation of the linear portion of the saturated ion current characteristics of the highly negative probe to less negative voltages. The electron current is then obtained by subtracting the positive ion current, as is given by the extrapolated line, from the total current.

3.1 THE CONCEPT AND LOCATION OF SPACE POTENTIAL

It is in the region of space potential that differences between actual probe characteristics and ideal characteristics are found even in well-behaved plasmas. The theory predicts that a graph of $\ln i_e$ vs. V obtained from the characteristics of probes of any shape should break away from linearity when the probe potential just exceeds space potential. In practice a sharp break is rarely obtained and a rounding of the 'knee' is observed. In an attempt to explain this apparent discrepancy it is desirable to discuss the concept of space potential.

That the property of a potential may be ascribed to a region of plasma implies that no macroscopic electric fields exist throughout the region either due to the external application of such fields or due to charge separation as a result of charged particle density gradients within the region. The situation whereby a probe could be made to assume the same potential as the surrounding region could be approximated to, provided that the potential drop in the plasma across a distance equivalent to the largest dimension of the probe as a result of such a field is negligibly small. However, the fact that a probe at this potential will draw current from the surrounding plasma will result in the formation of density gradients in the region adjacent to the probe. For this perturbation to be minimized it is further required that the current to the probe must be sufficiently small to ensure that the influx of charged particles from the body of the plasma to the region near to the probe is great enough to prevent the formation of appreciable electric fields in the region. Because of this the probe potential may be made only to approximate to the potential of the surrounding plasma, the degree of approximation depending on the conditions peculiar to the particular plasma probe system.

Other reasons have also been suggested to explain the discrepancy between ideal and actual probe characteristics in the region of space potential. The three main ones are the reflection of electrons at the probe surface (VAN VOORHIS and COMPTON, 1930; LAMAR and COMPTON, 1931), secondary electron emission at the probe surface due to the action of metastable atoms (FOUND, 1929; UTERHOEVEN and HARRINGTON, 1930; LANGMUIR and FOUND, 1930), positive ions (OLIPHANT, 1929, 1931; UTERHOEVEN and HARRINGTON, 1930) and photons (KENTY, 1931), and finally the presence of insulating layers on the probe surface due to contaminants (EASLEY, 1951; WEHNER and MEDICUS, 1952). Because of these contributing factors, the location of space potential has become a matter of definition and compromise. Some workers still prefer to define space potential as that at which ln i_e vs. V plots deviate from linearity (HOYAUX, 1954), others as that given by the intersection point of the tangent drawn to the curve above the knee with the extrapolation of the linear portion below the knee (DRUYVESTEYN, 1930). An alternative method used is that suggested by a consideration of equation (5), space potential being kT_e/e V greater than the intersection point of the i_e^2 vs. V line with the voltage axis. Space potential has also been defined as the potential at which the second derivative of the probe current with respect to probe voltage vanishes (SLOANE and MACGREGOR, 1934) in order to make probe measurements of electron energy distributions meaningful, though it has also been suggested that space potential is that at which this derivative is a maximum (VOROB'EVA *et. al.*, 1963). A more recent method derives from the fact that the dynamic response of a probe changes significantly as it is pulsed through space potential (BILLS *et al.*, 1962).

3.2. DETERMINATION OF ELECTRON DENSITY

In general, values of the electron density are calculated from the current flowing to the probe at space potential [see equation (2)], the determination necessarily relying on the location of space potential. In addition to those inferred above there are two further methods for obtaining electron density values from cylindrical probe characteristics. One, which has already been mentioned, is that given by the slope of the i_e^2 vs. V graph for accelerating potentials. The other suggests that, although the probe can assume space potential, it does not collect the full current predicted by simple theory because of the reflection of electrons at the probe surface (LAMAR and COMPTON, 1931). In accordance with this, the current that would be collected at space potential in the absence of reflections is defined as that given by the point at which the tangents intersect.

Little work has been done in correlating the values of electron densities obtained by employing the above methods, though HOYAUX (1954) has compared three different methods of determining space potential from experiments made in a mercury arc discharge. He found that if either (a) the voltage range over which the accelerating characteristics were examined was not sufficiently large when a/r is large or (b) a/r could not be considered to be large, then the value of space potential as determined from an i_e^2 vs. V plot will fortuitously be in close agreement with that obtained from the method of intersecting tangents. He concluded that space potential is that given by the break-away point.

Several workers have reported attempts to compare electron densities as determined from Langmuir probe characteristics with those obtained using independent techniques (SCHULTZ and BROWN, 1955; YEUNG and SAYERS, 1957; TALBOT *et al.*, 1963; SMITH and OSBORNE, 1966; RUSBRIDGE and WORT, 1967; NICOLL and BASU, 1962; VERMA and POLISHUK, 1967). Not all of these workers, however, have indicated the method used to obtain the electron densities from the characteristics. Of those that do, NICOLL and BASU (1962) are in agreement with Hoyaux in so far as good correlation was found to exist between microwave determinations of electron densities in a low-pressure mercury-vapour arc and those obtained from probe measurements using the break-away point. VERMA and POLISHUK (1967), on the other hand, find a close correspondence between their microwave measurements in the positive column of a glow discharge in argon and probe determinations using the intersecting tangents method. Although the detailed agreement is seemingly not so convincing, their results suggest that a better correlation would exist if space potential were given by a point on the characteristics more positive than that given by the intersecting tangents method. However, in both experiments reported above, the spatial distribution of electron density across the plasma columns investigated were non-uniform, so that an average value for the electron density over the region examined had to be computed from the local probe measurements at various radial positions in order that a comparison with the microwave measurements could be made. In this context the comparisons cannot be considered direct.

4. USE OF LANGMUIR PROBES IN AFTERGLOW PLASMAS

It can be seen from equations (1) and (5) that in the retarding and orbital limited regions of the probe characteristics the electron current to a probe at a fixed potential is directly proportional to the undisturbed electron number density in the plasma. It is this additional property of the behaviour of probes that has also been exploited in the present studies of afterglow plasmas, thus providing a simple system for time resolved studies to be made of electron loss processes from the volume of the plasma. For sensible measurements to be made however, the current drawn to the probe must only represent a rate of loss of electrons that is insignificant in comparison with the undisturbed loss rate of the electrons from the whole volume of the plasma (COPSEY, 1964). This criterion places an upper limit on the size of probe that may be used in afterglow plasmas without serious depletion taking place.

Provided that sufficiently small probes are used, the afterglow plasma bounded by a large vessel provides an excellent system for the study of probe behaviour and since only ambipolar fields are present, the energy distribution of the charged particles probably represents the best approximation to a Maxwellian distribution. This is especially so in the recombination controlled afterglow where the charged particle density distribution is also expected to be isotropic and where any ambipolar fields will be small.

Further advantages in using afterglow plasmas (except under conditions of extremely high electron densities) as opposed to d.c. discharges lie in the facts that the complete probe characteristics may be examined without the effects of probe heating (CHEN, 1964; WILLS, 1967), secondary electron emission from the probe surface, or ionization within the probe sheath (LANGMUIR and MOTT-SMITH 1923, 1924).

5. EXPERIMENTAL APPARATUS

The observations were made in afterglows following r.f. pulsed discharges, the pulses being of 10 μsec duration at a frequency of 10 Mc/sec. The repetition frequency of the pulses could be varied up to 50 pulses/sec and the power in the pulse was continuously variable up to a maximum of approximately 100 kVA. The r.f. pulses were coupled into the discharge vessel through external sleeve electrodes. The discharge vessel was a Bluesil glass cylinder 23 cm long and 15·4 cm i.d. Two internal nickel electrodes having a total area of 150 cm^2 were used as reference electrodes for the probe system and enabled plasma potential to be controlled. The complete probe assemblies were situated in side arms and could be moved by means of external

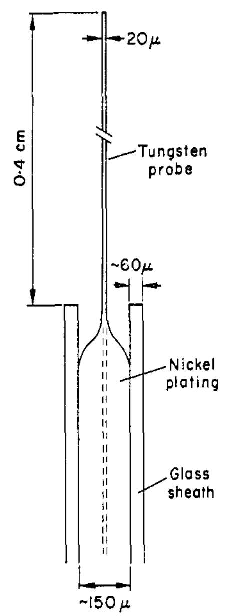

FIG. 1.—Enlarged diagram showing probe construction.

magnets so that the probe tip could be made to occupy a variety of positions in the discharge volume.

Standard vacuum techniques were employed to evacuate the discharge vessel to a residual pressure of about 10^{-7} torr.

A cylindrical probe was chosen for the present series of experiments in preference to the spherical or plane probe because of its comparative ease of construction relative to its design requirements. Tungsten was chosen as the probe material to minimize sputtering effects. The essential features of the probe and its support are shown in Fig. 1. Tungsten wire of 2×10^{-3} cm dia. was nickel plated to a diameter of about $1{\cdot}5 \times 10^{-2}$ cm. A short length of the tungsten was then exposed by preferential etching of the nickel. In this way the nickel acted as a collar and contained the probe concentric with a thin pyrex glass sheath about 10 cm in length which had been drawn down until its internal diameter was the same as the diameter of the nickel-plated tungsten. The probe was positioned in the glass sheath giving a probe collecting area of $2{\cdot}5 \times 10^{-3}$ cm^2. These design features ensured that the effective collecting area of the probe could not be increased by its being in contact with a film of sputtered metal on the glass surface and that the influence of the glass surface on the probe behaviour could be minimized.

6. THE PROBE CIRCUITRY

The current–voltage characteristics of the probe were automatically obtained by monitoring the signal developed across a resistor in the probe–reference electrode system while a negative going ramp voltage was applied to the reference electrodes immersed in the plasma. Filter circuits protected both the ramp voltage generator and the probe signal amplifier from r.f. pulses picked up by the reference electrodes and the probe.

The probe signal developed across a range of resistances up to a maximum value of 1 kΩ was amplified by a two-stage transistorized amplifier (gain $\sim$80) with a zero d.c. level output. Gating techniques (WAGER, 1960) were employed to ground the amplifier probe signal for any predetermined time after the initiation of the discharge so that late afterglow signals could be further amplified without saturating the oscilloscope amplifiers. The trailing edge of the gate was used to trigger a square-wave generator which could provide a synchronous or delayed pulse of variable duration and magnitude. This was applied to the Z-axis input of a Hewlett–Packard oscilloscope (140 A) thus intensifying the scope trace at the time of interest. A second output derived from the ramp voltage generator was applied to the horizontal input of the oscilloscope. In this way the current–voltage characteristics of the probe at a fixed time in the afterglow were displayed on the oscilloscope screen by a series of dots separated in time by an amount determined by the repetition frequency of the r.f. discharge pulses.

To obtain an accurate determination of the electron temperature from the retarding characteristics of the probe, the positive ion current was subtracted from the total probe current. This is normally done graphically. However, by applying a suitably attenuated portion of the output from the ramp voltage generator to the second channel of the dual trace amplifier and displaying the difference between the two input signals, the extrapolation and subtraction technique described in Section 2 was performed automatically, the difference between the dotted trace and a horizontal reference line representing the electron current to the probe only. Measurements were extracted from photographs taken of the oscillograms.

By automatically obtaining the current–voltage characteristics of the probe the effect of changing contact potential differences during the observation time ($\sim$1 sec) could be minimized. In addition to this, the probe was cleaned prior to observations being taken by heating it, this being accomplished by the action of electron bombardment under the application of a large positive voltage to the probe with respect to the plasma.

7. RESULTS

Probe characteristics were obtained in afterglows produced in helium, argon, helium–oxygen and argon–oxygen mixtures in a discharge vessel at room temperature. The probe was positioned in the centre of the discharge volume so that even in the diffusion controlled afterglow the expected variation in electron density along the probe length was negligibly small. A total of 33 probe characteristics were examined over a measured electron temperature range 600–900°K and electron density range 10^9–10^{11}/cm^3. Under these conditions the electron current to the probe at accelerating potentials was considered to be orbital limited. For example, when $T_e = 600$°K, $n_e = 10^{10}$ electrons/cm^3 and $\eta = 4$ (this value of η corresponding to a probe voltage of 0·2 V positive with respect to space potential), ϕ equals 5×10^{-2} or equivalently a/r equals 10. Since $(a^2 - r^2)$ increases more rapidly than η increases, ϕ decreases for increasing values of η, so the orbital limited current conditions were satisfied over the voltage range studied. This was verified by the linearity of the

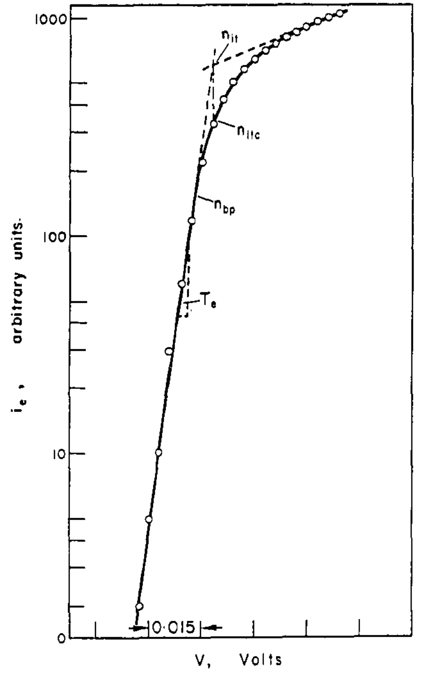

FIG. 2.—Typical probe characteristics for electrons.

i_e^2 vs. V plots obtained from the accelerating characteristics. Typical ln i_e vs. V and i_e^2 vs. V plots are shown in Figs. 2 and 3 respectively. The i_e^2 vs. V plots were found to be linear over an equivalent range of up to $40kT_e$ in voltage, this being the maximum, useful voltage range necessary for this study.

The measured electron temperatures were always found to be two to three times greater than the ambient temperature of the containing walls of the afterglow. On

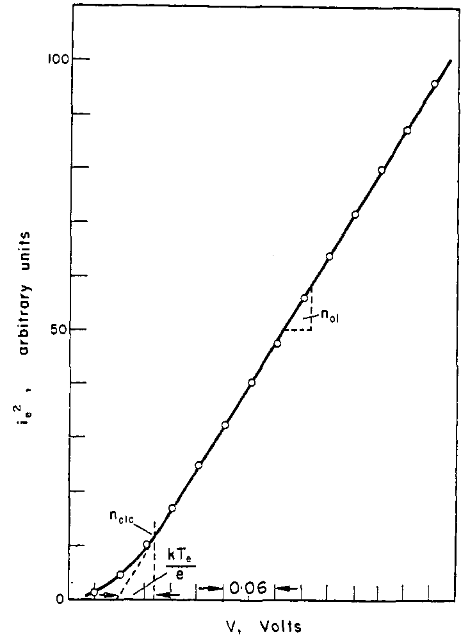

FIG. 3.—Typical orbital limited characteristics for electrons.

initial examination these findings are unexpected since it may be thought that the electrons would be in thermal equilibrium with the walls of the vessel via collisions with neutral gas atoms within a few hundred microseconds following the termination of the discharge pulse (OSKAM, 1958). However, the present results are not in disagreement with those reported by some other workers (MOSBERG, 1966; KAPLAFKA and GOLDSTEIN, 1967). Super-elastic collisions of electrons with metastable atoms (PHELPS, 1955; FERGUSON and SCHLÜTER, 1962) and metastable–metastable collisions (PHELPS and MOLNAR, 1953) may be contributing factors, but there is also experimental evidence, recently obtained with independent techniques, which suggests that the neutral gas temperature itself could be enhanced in the centre of the afterglow and maintained above wall temperature for periods greater than a millisecond depending on the gas pressure and container geometry (GERARDO *et al.*, 1965; GUSINOW *et al.*, 1966; BORN and BUSER, 1966). A more detailed analysis of the present probe measurements of electron temperatures in afterglows will be made in a subsequent paper (SMITH *et al.*, to be published).

Electron densities were determined from the experimentally obtained characteristics by six different methods as shown below, the first five of which assume that the probe collects the full predicted current at space potential [see equation (2)]. The associated point on the characteristics corresponding to the electron density values obtained using these methods are indicated in Figs. 2 and 3, the notation adopted being:

n_{bp} assuming space potential to be given by the break-away point;

n_{0l} obtained from the slope of an i_e^2 vs. V plot;

n_{dd} assuming space potential to be given by the voltage at which $\mathrm{d}^2i/\mathrm{d}V^2 = 0$;

n_{0lc} assuming space potential to be kT_e/e volts greater than the intercept on the voltage axis of an i_e^2 vs. V plot;

n_{itc} assuming space potential to be given by the intersecting tangents method;

n_{it} assuming that both space potential and the probe current at space potential is given by the loci of the intersecting tangents.

Since the characteristics were not continuous in that they were composed of signals taken in successive afterglows, electronic methods of double differentiation could not be used. Instead graphical methods were employed to determine the point of inflexion of the characteristics. This was done by finding the voltage range over which the i–V curves were linear and then assuming the inflexion to be that given by the mean voltage over this range. In a number of cases this was then checked by obtaining the complete energy distribution curve using the graphical method of MEDICUS (1956) and determining the electron temperature given by the distribution. Electron temperatures obtained in this way were found to be in close agreement with those given by the slope of the corresponding ln i_e vs. V plots. The potentials at which $\mathrm{d}^2i/\mathrm{d}V^2$ was a maximum were not investigated since these clearly correspond to those given by the break-away point.

In order to investigate any measure of agreement between the above methods of determining the electron density, the quotients of the values of electron density obtained using the various methods were calculated. The average quotients of the 33 sets of readings taken were:

$\frac{n_{0l}}{n_{bp}} = 2{\cdot}53\ (\pm 0{\cdot}13)$

$\frac{n_{dd}}{n_{bp}} = 2{\cdot}62\ (\pm 0{\cdot}15)$ $\quad \frac{n_{dd}}{n_{0l}} = 1{\cdot}03\ (\pm 0{\cdot}02)$

$\frac{n_{0lc}}{n_{bp}} = 2{\cdot}85\ (\pm 0{\cdot}16)$ $\quad \frac{n_{0lc}}{n_{0l}} = 1{\cdot}13\ (\pm 0{\cdot}01)$ $\quad \frac{n_{0lc}}{n_{dd}} = 1{\cdot}10\ (\pm 0{\cdot}02)$

$\frac{n_{itc}}{n_{bp}} = 2{\cdot}62\ (\pm 0{\cdot}10)$ $\quad \frac{n_{itc}}{n_{0l}} = 1{\cdot}07\ (\pm 0{\cdot}03)$ $\quad \frac{n_{itc}}{n_{dd}} = 1{\cdot}06\ (\pm 0{\cdot}04)$ $\quad \frac{n_{itc}}{n_{0lc}} = 0{\cdot}96\ (\pm 0{\cdot}03)$

$\frac{n_{it}}{n_{bp}} = 4{\cdot}36\ (\pm 0{\cdot}19)$ $\quad \frac{n_{it}}{n_{0l}} = 1{\cdot}77\ (\pm 0{\cdot}05)$ $\quad \frac{n_{it}}{n_{dd}} = 1{\cdot}74\ (\pm 0{\cdot}06)$ $\quad \frac{n_{it}}{n_{0lc}} = 1{\cdot}57\ (\pm 0{\cdot}04)$ $\quad \frac{n_{it}}{n_{itc}} = 1{\cdot}65\ (\pm 0{\cdot}03)$

The errors given represent the standard deviations.

It can be seen from the results that the values given by n_{0l} and n_{dd} are in good agreement, their quotients being very close to unity, these also apparently being in reasonable agreement with n_{itc}. It must be noted, however, that the value of n_{itc} obtained will depend on the point at which the tangent is drawn to the semilogarithmic characteristics in the accelerating region. Under orbital limited current conditions this will then be solely determined by the voltage range over which the accelerating characteristics are observed. Any error introduced due to this will be more serious at high electron temperatures since the slope of the ln i_e vs. V plot will be smaller in the retarding region while the characteristics above space potential will remain unchanged [see equation (5)]. In this respect it is significant that the statistical errors associated with the quotients n_{itc}/n_{0l} and n_{itc}/n_{dd} are greater than those associated with n_{dd}/n_{0l}.

An interesting result is contained in the value obtained for n_{0lc}/n_{0l}, this being 1·13 with the small standard deviation of 0·01. This value is identical with $2/\sqrt{\pi}$ and is that of the dimensionless constant which is obtained from equation (5) when the voltage, V, is placed equal to zero and the probe current expressed in terms of the probe area, A, and the undisturbed current density, j_e, [cf. equation (2)]. This signifies that the i_e^2 vs. V relation holds even at the value of space potential predicted by the orbital limited equation and, in this respect, n_{0lc} cannot be considered as an independent value of the electron density. It is in the region of space potential that the orbital limited equation as stated should not apply (since $\eta \to 0$), the behaviour of the probe being explained by equation (3). In view of the good linearity of the i_e^2 vs. V plots this deviation from the theory is unexpected and as yet no explanation has been found, though some clarification may be obtained from observations made with an even smaller probe which it is proposed to use in future experiments.

The difference between n_{0l}, n_{dd} and n_{itc} and n_{bp} will depend on the electron temperature due to the uncertainty in locating the break-away point. This is indicated by the large errors associated with the quotients containing n_{bp}. The rather large value of these quotients of about 2·6 obtained in the present experiments is partly due to the low electron temperatures characteristic of afterglow plasmas. For higher electron temperatures this factor is therefore expected to decrease.

The electron densities obtained using the intersecting tangents method (n_{it}) are about a factor of 1·5 to 2 greater than n_{0l} and n_{dd}. But values of n_{it}, as for those of n_{itc}, will depend on the point at which the tangent to the accelerating characteristics is drawn, thus introducing a comparatively high degree of uncertainty, though in every case, n_{it} was found to be significantly larger than either n_{0l} or n_{dd}. This is indicated by the errors associated with the quotients n_{it}/n_{0l} and n_{it}/n_{dd} as compared with those of n_{it}/n_{itc}.

8. SUMMARY AND CONCLUSIONS

This work describes Langmuir probe measurements made in gaseous afterglow plasmas in a discharge vessel at room temperature and compares the values of electron densities inferred from the probe characteristics by the different methods which are currently used. These are further compared with values of electron density obtained from the orbital limited characteristics, a method which, although not so widely applicable due to the limitations given in Section 2, does avoid a number of serious difficulties associated with the other methods. It is not possible to state which of the methods yield the most appropriate values of electron density without recourse to simultaneous measurements using independent techniques in systems where direct comparison can be made. However, the findings do indicate that electron density values deduced from the orbital limited characteristics (n_{0l}) and from the probe current at the inflexion point of the characteristics (n_{dd}) are in close agreement. Further, they were found to lie approximately midway between the extremes, n_{bp} and n_{it}. These two facts may shed some light on the meaning or concept of space potential. The tolerable agreement also found between the values of n_{0l} and n_{dd} and those of n_{itc} is not considered significant or due, in any way, to an inherent coincidence as was found by HOYAUX (1954). However, such differences between the present work and those of others need not necessarily be conclusive since the factors governing the behaviour of probes in various types of plasma could be fundamentally different.

In spite of the close correspondence between the respective values of n_{dd} and n_{0l}, the overall spread in values using the different methods is large, this being due to the large divergence in the values of n_{bp} and n_{it}. These were found to differ by a factor of about 5. Since values of n_{it} are determined by the intersecting tangents method which assumes the occurrence of electron reflections at the probe surface and is not so widely used as the other, a more representative and realistic factor would be about 3. Both factors, however, may be expected to decrease in situations where greater electron temperatures exist.

The advantages in using the orbital limited characteristics are threefold. Measurements of electron densities rely on the determination of a line rather than on the location of a point, no values of electron temperature are required, and any slight fluctuations in space potential produce only minimal changes in the current and hence in the slope of the i_e^2 vs. V plots. These advantages also apply in part to the use of spherical probes where theory predicts a linear i–V relationship in the orbital limited region, except that absolute determinations of electron densities from these characteristics require a knowledge of the electron temperature. However, this functional difference in the respective characteristics could itself be exploited in determining electron temperatures by using a two-probe system; one probe being cylindrical and the other spherical.

For the reasons given above, a cylindrical probe used in the orbital limited region is considered by the authors to be a useful diagnostic tool not only in the study of ion–electron loss processes occurring in afterglow plasmas but also in other situations, notably the ionosphere, where the conditions are expected to be conductive to its application.

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REFERENCES

BETTINGER R. T. and WALKER E. H. (1965) *Physics Fluids* **8**, 748.
BILLS D. G., HOLT R. B. and MCCLURE B. T. (1962) *J. appl. Phys.* **33**, 29.
BORN G. K. and BUSER R. G. (1966) *J. appl. Phys.* **37**, 4918.
BOYD R. L. F. and TWIDDY N. D. (1959) *Proc. R. Soc.* **A250**, 53.
CHEN C. J. (1964) *J. appl. Phys.* **35**, 1130.
COPSEY M. J. (1964) Ph.D. Thesis, Birmingham University.
DEBYE P. and HÜCKEL E. (1923) *Phys. Z.* **24**, 185, 305.
DRUYVESTEYN M. J. (1930) *Z. Phys.* **64**, 781.
EASLEY M. A. (1951) *J. appl. Phys.* **22**, 590.

FERGUSON E. E. and SCHLÜTER H. (1962) *Planet. Space Sci.* **9,** 701.
FOUND C. G. (1929) *Phys. Rev.* **34,** 1625.
GERARDO J. B., VERDEYEN J. T. and GUSINOW M. A. (1965) *J. appl. Phys.* **36,** 3526.
GUSINOW M. A., GERARDO J. B. and VERDEYEN J. T. (1966) *Phys. Rev.* **149,** 91.
HOYAUX M. (1954) *Direct Current* **1-2,** 22.
JOHNSON E. O. and MALTER L. (1950) *Phys. Rev.* **80,** 58.
KAPLAFKA J. and GOLDSTEIN L. (1967) N.A.S.A. Report NGR 14-005-037.
KENTY C. (1931) *Phys. Rev.* **38,** 377, 2079.
LAMAR E. S. and COMPTON K. T. (1930) *Phys. Rev.* **37,** 1069.
LANGMUIR I. (1913) *Phys. Rev.* **2,** 450.
LANGMUIR I. and BLODGETT K. B. (1923) *Phys. Rev.* **22,** 347.
LANGMUIR I. and COMPTON K. T. (1931) *Rev. mod. Phys.* **3,** 214.
LANGMUIR I. and FOUND C. G. (1930) *Phys. Rev.* **36,** 604.
LANGMUIR I. and MOTT-SMITH, H. M. (1923) *Gen. Elect. Rev.* **26,** 731.
LANGMUIR I. and MOTT-SMITH H. M. (1924) *Gen. Elect. Rev.* **27,** 7, 449, 616.
LECKEY R. C. G., COULTER A. G., HIGGINSON G. S. and WEBB T. G. (1963) *Proc. phys. Soc.* **82,** 947.
LOEB L. B. (1960) *Basic Processes of Gaseous Electronics*, 2nd edn., p. 361, U.C.P.
MCDANIEL E. W. (1964) *Collision Phenomena in Ionized Gases*, p. 693, Wiley.
MEDICUS G. (1956) *J. appl. Phys.* **27,** 1242.
MOSBERG E. R. (1966) *Phys. Rev.* **152,** 166.
MOTT-SMITH H. M. and LANGMUIR I. (1926) *Phys. Rev.* **28,** 727.
NICOLL G. R. and BASU J. (1962) *J. Electron. Control* **12,** 23.
OLIPHANT M. L. E. (1929) *Proc. R. Soc.* **A124,** 228.
OLIPHANT M. L. E. (1931) *Proc. R. Soc.* **A132,** 631.
OSKAM H. J. (1958) *Philips Res. Rept.* **13,** 367.
PHELPS A. V. (1955) *Phys. Rev.* **99,** 1307.
PHELPS A. V. and MOLNAR J. P. (1953) *Phys. Rev.* **89,** 1202.
RUSBRIDGE M. G. and WORT D. J. H. (1967) *Plasma Phys.* **9,** 239.
SCHULTZ G. J. and BROWN S. C. (1955) *Phys. Rev.* **98,** 1642.
SLOANE R. H. and MACGREGOR E. I. R. (1934) *Phil. Mag.* **18,** 193.
SMITH D., GOODALL C. V. and COPSEY M. J. To be published.
SMITH F. H. C. and OSBORNE F. J. F. (1966) R.C.A. Victor Res. Rept. 7-801-47.
TALBOT L., KATZ J. and BRUNDIN M. (1963) *Physics Fluids* **6,** 559.
UTERHOEVEN W. V. and HARRINGTON M. C. (1930) *Phys. Rev.* **35,** 124, 438, 1421; **36,** 709.
VAN ECK J. and KINDERDIJK H. M. J. (1967) *Physica* **33,** 679.
VAN VOORHIS C. C. (1927) *Phys. Rev.* **30,** 318.
VAN VOORHIS C. C. and COMPTON K. T. (1930) *Phys. Rev.* **36,** 1435.
VERMA J. S. and POLISHUK P. (1967) *Bull. Am. phys. Soc.* **12,** 192.
VOROB'EVA N. A., KAGAN YU. M. and MILENIN V. M. (1963) *Soviet Phys. tech. Phys.* **8,** 423.
WAGER J. H. (1960) *J. Electron. Control* **8,** 227.
WALKER E. H. (1965) *Interaction of Space Vehicles with an Ionized Atmosphere* (S. F. SINGER, Ed.) Pergamon Press.
WEHNER G. and MEDICUS G. (1952) *J. appl. Phys.* **23,** 1035.
WILLS T. G. (1967) R.A.E. Tech. Rept. 67036.
YEUNG T. H. Y. and SAYERS J. (1951) *Proc. phys. Soc.* **B70,** 663.