

# Electron mobility estimation of hollow cathode plasma in different electrical configurations

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**In Hall effect thrusters (HETs), several experimental works suggest that the thruster performance (e.g., discharge current oscillation) is affected by the electrical configurations of the thruster body. Recent measurement data of the electron density  $n_e$  and electron temperature  $T_e$  obtained from the incoherent laser Thomson scattering show that the near-field hollow cathode discharge is also affected depending on the electrical configuration in a center-mounted HET. In this study, the measurement data of  $n_e$  and  $T_e$  are input to a one-dimensional fluid model, and the electron mobility is estimated for three electrical configurations: ground, floating, and cathode-tied. The estimated results, such as the anomalous electron scattering frequency, agree with the literature. In addition, the estimated results suggest that the electron mobility is the highest for the ground condition, while the electron conductivity is comparable between the floating and cathode-tied conditions.**

## I. Introduction

Electric propulsion (EP) devices have been tested in ground facilities (i.e., a vacuum chamber) for predicting thruster performance (e.g., thrust and lifetime). However, several in-space operations have clarified the difference in thruster performance between ground tests and space operations [1–4], which is attributed to the so-called facility effect. Facility effects are mainly divided into three problems [5]: background pressure, contamination, and electrical facility effects.

In this paper, we focus on the electrically facility effect in Hall effect thrusters (HETs). Figure 1 shows three electrical configurations of the thruster body in HET. For the ground configuration, the thruster body is connected to the chamber ground. For the floating configuration, the thruster is electrically isolated from the chamber ground. Finally, for the cathode-tied configuration, the thruster voltage is equal to the cathode voltage relative to the ground,  $V_{c,g}$ . Several works suggest that thrust [6], the discharge current [6–8], and the discharge current oscillations [6, 8–10] are affected by the electrical configurations. In addition, a recent study shows the electron properties (e.g., electron density and temperature) at near-field hollow cathode discharge are also modified by the electrical configuration, as measured by incoherent laser Thomson scattering (LTS) [10]. In this work, the experimental data of the electron density and temperature is input into a one-dimensional fluid model, and then the electrical mobility is estimated in different electrical configurations.

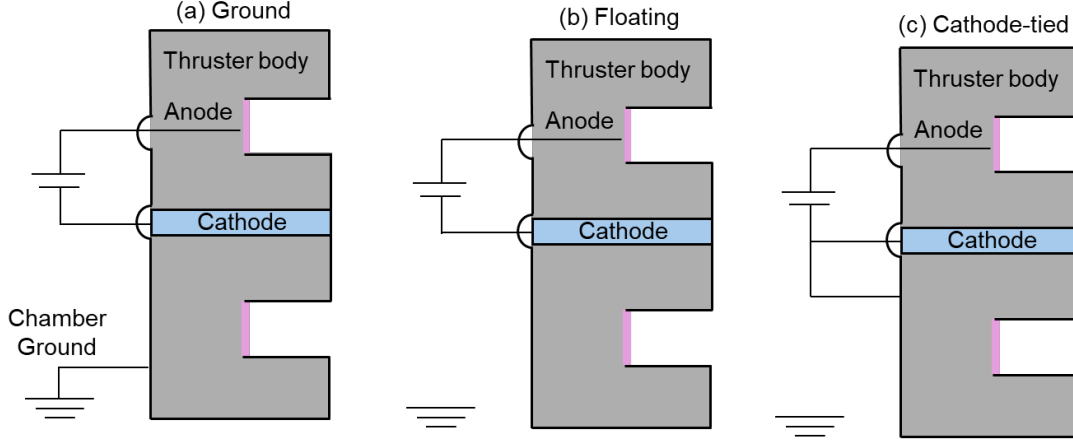
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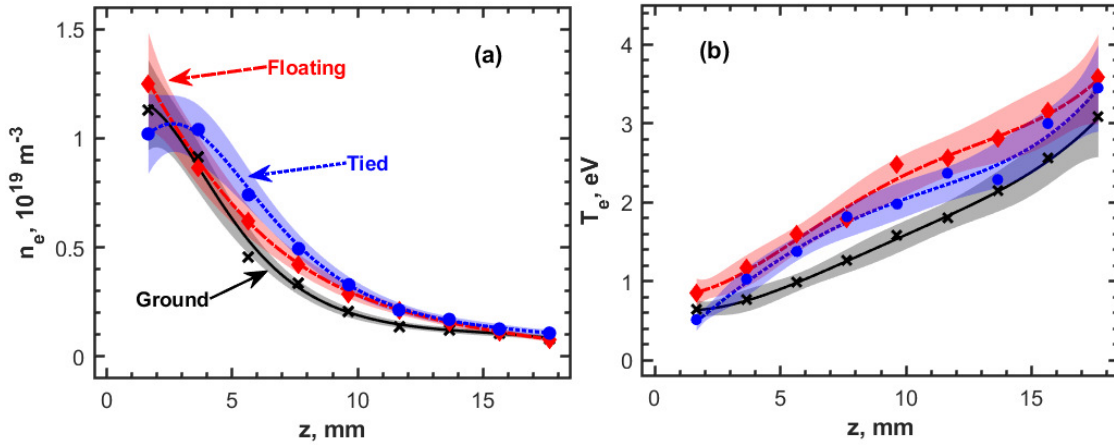
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**Fig. 1** Electrical configuration of the thruster body in HET. The testing was performed with three different electrical configurations: (a) ground, (b) floating, and (c) cathode tied.

## II. Experimental condition and data

In this paper, experimental data of the hollow cathode plume are used [10]. The H9 thruster is operated with an input power of 6 kW, where the discharge voltage is  $V_d = 300$  V and the discharge current is  $I_d = 20$  A, in the VTF-2 vacuum chamber at Georgia Institute of Technology. A hollow cathode is mounted at the center of the HET. The working gas is krypton, and the mass flow at the cathode  $\dot{m}_c$  is 1.26 mg/s. Figure 2 shows (a)  $n_e$  and (b)  $T_e$  for three electrical configurations. Symbols show the experimental data of  $n_e$  and  $T_e$  obtained from the LTS measurement. In this study, to substitute the experimental data into a one-dimensional fluid model, polynomial fitting is performed. In addition, the experimental uncertainty is assumed to be 15% for both  $n_e$  and  $T_e$ . That is, each data point is generated considering the experimental uncertainty using a random number, and then polynomial fitting is performed. By repeating this procedure many times (e.g.,  $10^4$ ), the mean value and standard deviation  $\sigma$  are evaluated for  $n_e$  and  $T_e$ . In Fig. 2, the solid line shows the mean value, and the shadow region shows  $2\sigma$ , respectively.



**Fig. 2** (a).  $n_e$ , (b)  $T_e$  in three electrical configurations: ground, floating, and tied. Symbol: experimental data obtained from the incoherent laser Thomson scattering [10]. Solid line: mean value of the polynomial fitting. Shadow region:  $2\sigma$  of the polynomial fitting. In this study,  $\pm 15\%$  uncertainty of experimental data is considered.

### III. Estimation of electron mobility

The effective electron mobility  $\mu_{\text{eff}}$  is defined as,

$$n_e u_{e,z} = -\mu_{\text{eff}} \left( n_e E_z + \frac{1}{e} \frac{\partial p_e}{\partial z} \right). \quad (1)$$

where  $n_e$  is electron density,  $u_{e,z}$  is axial electron bulk velocity,  $E_z$  is axial electric field,  $e$  is elementary charge, and  $p_e = n_e k_B T_e$  is electron pressure ( $k_B$ : Boltzmann constant and  $T_e$  is electron temperature).  $\mu_{\text{eff}}$  is defined as,

$$\mu_{\text{eff}} = \frac{e}{m_e (\nu_m + \nu_{\text{ano}})}, \quad (2)$$

where  $\nu_m$  is the momentum transfer collision frequency, and  $\nu_{\text{ano}}$  is anomalous scattering frequency. Since LTS measurements provide  $n_e$  and  $T_e$ ,  $E_z$  and  $u_{e,z}$  are estimated using quasi-1D, steady-state, fluid models. Here, we use the neutral continuity, ion continuity, ion momentum equations, and current conservation [11].

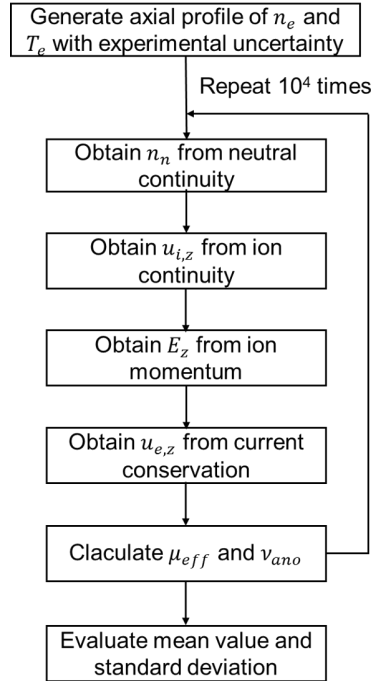
$$u_{n,z} \frac{dn_n}{dz} = -k_{iz} n_e n_n - \frac{u_{n,r}}{\Delta R} n_n, \quad (3)$$

$$\frac{d(n_i u_{i,z})}{dz} = k_{iz} n_e n_n - \frac{u_{i,r}}{\Delta R} n_n. \quad (4)$$

$$m_i \frac{d(n_i u_{i,z}^2)}{dz} + e T_i \frac{dn_i}{dz} = e n_i E_z + m_i k_{iz} n_e n_n u_{n,z} - m_i \frac{u_{i,r}}{\Delta R} n_i u_{i,z}. \quad (5)$$

$$I_d = e n_i (u_{i,z} - u_{e,z}) A. \quad (6)$$

where  $n_n$  is neutral density,  $u_{n,z}$  and  $u_{n,r}$  are axial and radial neutral velocities which assume a constant thermal velocity,  $k_{iz}$  is the rate coefficient of ionization which is function of  $T_e$  if assuming Maxwellian distribution,  $n_i$  is ion density which assumes quasi-neutral ( $n_i \approx n_e$ ),  $u_{i,z}$  and  $u_{i,r}$  are axial and radial ion velocities,  $\Delta R$  is the characteristics length of ambipolar type diffusion,  $I_d$  is the discharge current,  $u_{e,z}$  is axial ion velocity, and  $A$  is the surface area.



**Fig. 3** Flow chart for estimation of electron mobility from the experimental data of  $n_e$  and  $T_e$ . Uncertainty quantification is performed considering the uncertainty of experimental data and assumptions.

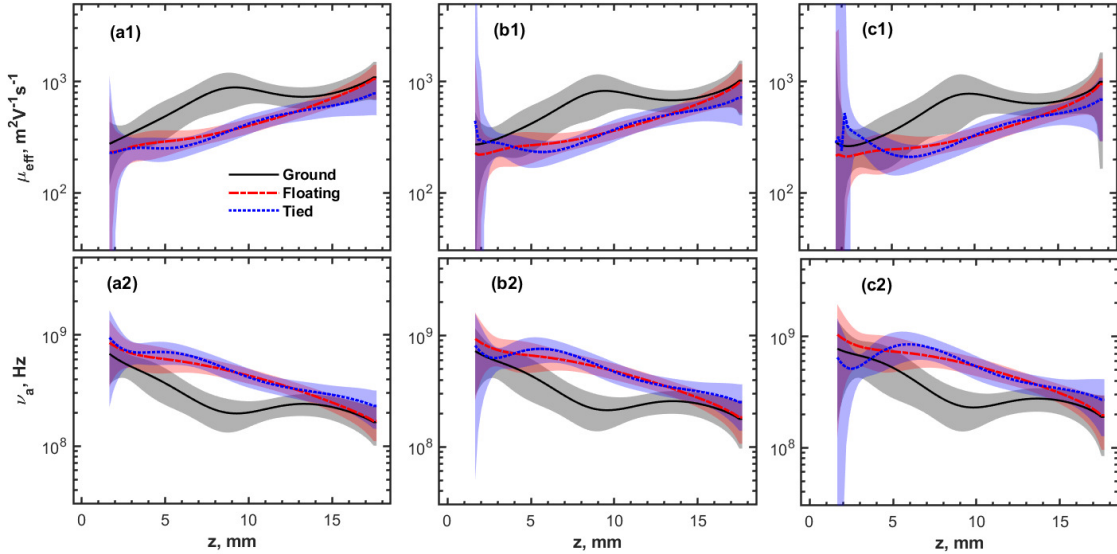
Figure 3 shows the flow chart for the estimation of  $\mu_{\text{eff}}$  and  $\nu_{\text{ano}}$ . In this case, since  $n_e$  and  $T_e$  are provided from the experiment, data assimilation techniques, such as Kalman filters [12], are not required. First,  $n_n$  is estimated from Eq.(3). Second,  $u_{i,z}$  is estimated from Eq.(4). Third,  $E_z$  is estimated from Eq.(5). Fourth,  $u_{e,z}$  is estimated from Eq.(6). Then,  $\mu_{\text{eff}}$  and  $\nu_{\text{ano}}$  are obtained from Eqs. (1) and (2). It should be noted that  $n_n(z=0) = 0.8\dot{m}_c/(Au_{n,z})$  is used when solving Eq.(3), and the ion bulk velocity at the plume is specified when solving Eq. (4).

Uncertainty quantification is performed to evaluate the experimental uncertainty and the assumed uncertainty. In this study, the experimental uncertainty is assumed to be 15% for both  $n_e$  and  $T_e$ . In addition, for the assumed uncertainty,  $T_n = 500 - 1500$  K,  $u_{i,zp} = 2000 - 3000$  m/s,  $T_i = 0.1 - 1.0$  eV are used. Each parameter is generated randomly between the range shown above, and then the mean value and standard deviation of  $\mu_{\text{eff}}$  and  $\nu_{\text{ano}}$  are estimated.

#### IV. Estimated results

Figure 4 shows the estimated  $\mu_{\text{eff}}$  and classical mobility  $\nu_{\text{ano}}$  in three electrical configurations using three values of  $\Delta R = 5, 6, 7$  mm. For (a1),  $\Delta R = 5$  mm, it can be seen that  $\mu_{\text{eff}}$  is the order of  $10^2 - 10^3$   $\text{m}^2\text{V}^{-1}\text{s}^{-1}$  and  $\mu_{\text{eff}}$  is one or two orders of magnitude smaller than the classical mobility, indicating the presence of the anomalous collision frequency. Figure 4 (a2) indicates that  $\nu_{\text{ano}}$  is the order of  $10^8 - 10^9$  Hz, which is in good agreement with the literature [13–16]. Figures 4(a1), (b1), and (c1) suggest that  $\mu_{\text{eff}}$  is not changed too much for different  $\Delta R$ . Both  $|E_z|$  and  $u_{e,z}$  become small when using large  $\Delta R$ , and thus  $\mu_{\text{eff}}$  is not much sensitive to  $\Delta R$  from Eq. (1). It should be noted that there is a large uncertainty around  $z = 1.75$  mm. In some cases,  $|E_{\text{eff}}| = E_z + (en_e)^{-1}\partial p_e/\partial z$  is close to 0 and  $\mu_{\text{eff}}$  becomes large significantly, resulting in the large uncertainty. The improvement of the boundary condition around  $z = 1.75$  mm is reserved for future work.

Most interestingly,  $\mu_{\text{eff}}$  (i.e., electron conductivity) is the largest for the ground condition, while  $\mu_{\text{eff}}$  of the floating condition is comparable to the tied condition. Interpolation of  $\mu_{\text{eff}}$  for three electrical configurations is reserved for future work.



**Fig. 4** Estimated properties: (1).  $\mu_{\text{eff}}$  and (2).  $\nu_{\text{ano}}$  for three electrical configurations: ground, floating, and tied. Solid line: mean value, shadow regions:  $2\sigma$  including the uncertainty of the experimental data shown in Fig. 2 and the assumed uncertainty. (a).  $\Delta R = 5$  mm, (b).  $\Delta R = 6$  mm, and (c).  $\Delta R = 7$  mm.

#### V. Conclusion

In this paper, the electron mobility is estimated at a near-field hollow cathode discharge using a 1D fluid model based on measurement data of electron density and electron temperature obtained from incoherent laser Thomson scattering. The estimation is performed for three electrical configurations of the thruster body: ground, floating, and cathode-tied. The estimated results suggest that the electron conductivity is the highest for the ground configuration,

while the electron conductivity is comparable between the floating and cathode-tied conditions.

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