Flow Control and Measurement in Electric Propulsion Systems: Towards an AIAA Reference Standard

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Accurate control and measurement of propellant flow to a thruster is one of the most basic and fundamental requirements for operation of electric propulsion systems, whether they be in the laboratory or on flight spacecraft. Hence, it is important for the electric propulsion community to have a common understanding of typical methods for flow control and measurement. This paper addresses the topic of propellant flow primarily for the gaseous propellant systems which have dominated laboratory research and flight application over the last few decades, although other types of systems are also briefly discussed. While most flight systems have employed a type of pressure-fed flow restrictor for flow control, both thermal-based and pressure-based mass flow controllers are routinely used in laboratories. Fundamentals and theory of operation of these types of controllers are presented, along with sources of uncertainty associated with their use. Methods of calibration and recommendations for calibration processes are presented. Finally, details of

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uncertainty calculations are presented for some common calibration methods and for the linear fits to calibration data that are commonly used.

Nomenclature

a	=	capillary radius, m
A, B	=	fit coefficients
C	=	proportional constant
C_o	=	orifice coefficient
c_p	=	specific heat capacity, J/kg·K
F	=	thrust, N
g	=	gravitational constant, 9.81 m/s^2
I_{sp}	=	specific impulse, s
L	=	length, m
m	=	mass, kg
$\dot{m}$	=	mass flow rate, kg/s
M_w	=	molecular weight, AMU
p	=	pressure, Pa
P	=	power, W
R	=	universal gas constant, 8.314 kJ/kmol·K
Q	=	thermal energy, J
$\dot{Q}$	=	heat flow rate, W
t	=	time, s
T	=	temperature, K
u	=	velocity, m/s
v	=	voltage, V
V	=	volume, m^3
$\dot{w}$	=	flow throughput, Pa-m^3/s
γ	=	specific heat ratio
η	=	efficiency
ρ	=	gas density, kg/m^3
σ	=	uncertainty

I. Introduction

Recently an initiative was begun in the electric propulsion (EP) community to document standard techniques for test and measurement. These documents, which may take the forms of standards, recommended practices, or guides, include such areas as thrust measurement, flow measurement, and several types of probe diagnostics. As work in the field of EP continues the trend of advancing from the research laboratory to flight application, establishing communal standards becomes increasingly important.

Accurate control and measurement of propellant flow to a thruster is one of the most basic and fundamental requirements for operation and evaluation of EP systems, whether they be in the laboratory or on flight spacecraft. For example, mass flow rate appears directly in two important thruster performance parameters, the specific impulse and efficiency:

$$I_{sp} = \frac{F}{\dot{m}g} \tag{1}$$

$$\eta = \frac{1}{2}\frac{F^2}{\dot{m}P} \tag{2}$$

Hence, providing an accurate and repeatable flow in the laboratory is necessary for valid performance measurement and diagnostic activities, and is critical for understanding the total amount of propellant necessary to accomplish a desired spacecraft mission. Consequences of poor flow measurement include at worst the inability to complete a mission, but could also include premature thruster wear or deleterious spacecraft-plume interactions. Yet, flow measurement is not often given as much attention in the literature as other more difficult and interesting measurements, such as those involving plasma probes. Part of this is certainly due to the widespread availability and ease of use of accurate commercial products for gaseous flow control and measurement.

This paper will give a brief overview of flow measurement methods for EP systems, and focus in detail on the methods of gaseous flow measurement that are most widely applied in the laboratory and that are pertinent for recent and near-term flight application. It will describe the theory of operation of mass flow controllers, discuss calibration methods and techniques, and identify sources of error in their implementation. Because of the availability of mature, accurate, and easy-to-use commercial mass flow control products, a flow measurement reference document (or standard) is intended primarily as background information to make users aware of how these devices work and what sources of error are of typical concern in their use. Additionally, because of the wide variety of institutions that work in EP and their differing facilities and requirements, it may be appropriate for individual institutions to formulate their own procedures for flow measurement and control (e.g. for calibration).

The overarching goal of an AIAA standard for flow measurement is to provide a basis for the technological, calibration, and uncertainty discussions for flow mass controllers as pertaining to electric propulsion. With this basis in place, and with the understanding of the important impact that mass flow measurement has on thruster performance data, and thus system evaluation, all pursuant discussions of EP systems should include a description of the mass flow control system as it relates to that specific technology. The details discussed in this paper should be applied to technical presentations, journal articles, research reports, etc., as warranted.

II. Overview of Flow Measurement Types

A. Laboratory Systems

Most laboratory work with EP systems in the last few decades has focused on devices using propellants that are gaseous under typical storage and flow conditions, which are by far the most frequently used types of propellants in space application over the same time period. Many different gases are of interest for EP, for example those shown in Table 1, with xenon being the one that is predominantly used. Gaseous flow systems require two basic elements: a way to control flow rate and a way to measure (i.e. meter) that flow rate. The controlling and metering functions can either be combined into a single device or distributed separately within a flow system. In a distributed system, the controller could be, for example, a manually-actuated needle valve used in an open-loop fashion, or an automatic valve with closed-loop feedback from a separate meter. These types of systems can have advantages in terms of simplicity or cost, or they can provide advantages with respect to fault-tolerance. However, in recent years single-device mass flow controllers have advanced to the point where they are affordable, accurate, and easy to implement. Regardless of system type, flow metering is typically performed by either thermal-based or pressure-based methods, both of which will be discussed extensively in Section III.

Table 1. Examples of Gaseous Propellants of Interest for Electric Propulsion Systems.[1,2,3,4]

Noble Gases	Other Nonmetals	Compounds
Helium (He)	Hydrogen (H_2)	Ammonia (NH_3)
Neon (Ne)	Nitrogen (N_2)	Water Vapor (H_2O)
Argon (Ar)	Oxygen (O_2)	
Krypton (Kr)		
Xenon (Xe)		

EP devices other than gas-fed ion thrusters and Hall thrusters require very different types of flow measurement and control methods. For propellants that are solid under typical storage conditions, flow measurement typically involves measuring the mass of a propellant reservoir before and after a test to obtain an average flow rate. Liquid metal flow rates can be measured using electromagnetic flow sensors.[5,6] One of the more challenging aspects of micropropulsion systems is the development of new miniature components and methods for delivering solid, liquid, or gaseous propellants to a thruster. Colloid microthrusters can employ high-fidelity flow meters capable of measuring picoliters per second,[7] piezoelectric-actuated microvalves,[8] or use hydraulic impedance flow control[9] or time-of-flight methods.[10,11] With the exception of the flight-qualified colloid thruster system for the ST7-DRS mission,[8] however, these other types of EP systems are still in development and not yet in flight application, so in many instances flow measurement and control is an ongoing development and can be application specific. In these cases, development of a standard for such flow measurement systems would be premature.

B. Flight Systems

Nearly all xenon-based EP flight systems have controlled thruster mass flow rates using a pressure-fed flow restrictor.[12] For ion thruster systems, the flow regulation is typically achieved by closed-loop control of the pressure upstream of a calibrated flow restrictor. This type of system has the advantage of accurately controlling xenon flow at a known consumption rate, but at the cost of increased flow system complexity (e.g. inclusion of pressure transducers for each active flow branch). For Hall thruster systems, the flow control is typically not achieved by calibration of the flow restrictors, but rather through closed-loop control of the pressure based on measurement of the thruster discharge current. This type of system does not require the use of a pressure transducer for active flow control and thus can be much simpler, but does not yield knowledge of the actual xenon consumption rate.

The xenon feed system used on the NASA Deep Space 1 (DS1) and Dawn spacecraft is an example of a pressure-fed flow restrictor design used for ion thrusters.[12] For these two spacecraft, the pressure in a plenum tank is controlled via a "bang-bang" system (many other spacecraft use a mechanical regulator to control the pressure). The plenum pressure is measured by pressure transducers, and is then fed to flow control devices (FCDs) that are fitted with temperature transducers. Calibration of the FCDs was performed prior to spacecraft integration using a pressure blow-down method at several temperatures to generate mass flow rate curves as a function of pressure and temperature. Knowledge of the upstream pressure and FCD temperature in flight thus yields the mass flow rate.

Another method for sensing flow rates is based on the thermal properties of the propellant. This type of thermal flow sensor was used for the ion thrusters on the ESA GOCE spacecraft, where the cathode mass flow rates were controlled with pressure-fed FCDs, but mission requirements led to the use of an active control valve for regulating the main flow and a specially-developed thermal flow sensor to measure the main mass flow rate.[13] The JAXA ETS-VI and COMETS spacecraft used thermal mass flow controllers developed for space application derived from designs of those used within the semiconductor industry.[14]

For spacecraft which do not have a calibrated flow sensor, such as typical Hall thruster systems, propellant use must be estimated. There are three methods that are commonly used to determine the amount of propellant remaining onboard a spacecraft: Propellant Bookkeeping, Pressure-Volume-Temperature (PVT), and Thermal Mass Gauging.

1. Propellant Bookkeeping

The amount of propellant consumed by a thruster during a period of time can be estimated by multiplying the operational time of the thruster by its consumption rate. The propellant remaining onboard is then simply the initial amount minus the consumed amount. The accounting can be done on a "per-maneuver" basis:

$$m_{remaining} = m_{initial} - \sum_{i} \dot{m}_i \times \Delta t_i \tag{3}$$

Where m is the propellant mass, $\dot{m}_i$ is the mass flow rate for the i^{th} maneuver, and Δt_i is the duration of the i^{th} maneuver.

The initial amount of loaded propellant is known to a high degree of accuracy, based on the methods used to load a spacecraft (e.g. mass scales, high accuracy ground support equipment, etc.). The maneuver duration is often taken from the spacecraft telemetry, while the consumption rate may be measured directly with a mass flow sensor, measured indirectly with other fluid or electrical parameters, or it can be inferred based on ground tests at similar conditions. One complicating factor for the latter method for Hall thrusters is that the mass flow rate can vary over the course of a thruster lifetime because of how they are typically controlled.[15]

The bookkeeping method can be highly accurate over small ranges of time, but its accuracy suffers as the amount of mass consumed during the mission becomes large. The bookkeeping method relies on the accumulation of many separate calculations, thus any errors or inaccuracies in the calculation will also accumulate over time.

2. Pressure-volume-temperature

Using thermodynamic principles, the mass of a fluid can be determined when its pressure, volume, and temperature are known. This method is commonly known as the PVT method. Pressure and temperature can be taken from spacecraft telemetry, and the propellant tank volume is typically known from ground tests. Tank volumes, however, can change in response to pressure and temperature, so the appropriate correction factors must be applied to achieve an accurate result.

Depending on the type of propellant, other factors may need to be taken into consideration. Vapor pressure, phase changes, dissociation, and chemical reactions can all change the pressure inside the tank, and will affect the calculation. The behavior of xenon, in particular, is not well described by the ideal gas law for the typical conditions common to propulsion systems. An alternate description of the pressure-temperature-density relationship for xenon is necessary.

The great advantage of the PVT method compared to the bookkeeping method is that it is a point measurement. The result of a PVT calculation is independent of all previous measurements, thus errors do not accumulate. The accuracy of the PVT method is limited by the accuracy of the spacecraft sensors, although some improvement can be made by averaging several measurements together.

3. Thermal Mass Gauging

Thermal Mass Gauging relies on the inverse relationship between mass and temperature rise when heat is applied to a system:

$$m_{tank+propellant} = \frac{Q}{c_p \Delta T} \quad (4)$$

When a known amount of heat Q is applied to a propellant tank containing some amount of propellant the temperature of the system will increase. In a full tank the heat capacity of the propellant (propellant mass times propellant specific heat, c_p) will be much larger than the tank heat capacity (tank mass times tank specific heat), so the temperature rise will be much smaller than for a tank which is nearly empty. Similarly, if a known power is applied to a tank and propellant system, the temperature of a full tank will increase more slowly while a nearly-empty tank will increase more quickly. Hence, the propellant mass can be determined from the temperature change or rate-of-change after application of heater power. Additionally, a characterization can be done by removing heater power and observing the temperature decrease of the tank and propellant system.

Heat leakage from the propellant tank via conduction and radiation to the spacecraft must be accounted for with this method, but the associated uncertainties can be reduced by performing a thermal calibration test on the ground and through detailed system thermal modeling. All other things being equal, higher accuracy can be obtained from larger temperature changes. As the tank nears depletion and the propellant mass (and thus the thermal capacity) becomes relatively low, the temperature rise for a given amount of applied heat thus increases. This makes thermal mass gauging relatively more accurate near the end of the spacecraft mission, when the tank is close to depletion.

III. Fundamentals of Laboratory Measurement Devices

Government, commercial, and academic test facilities utilize commercial mass flow controllers (MFCs) to measure and control propellant into EP devices. To determine several key thruster performance parameters with a low level of uncertainty requires an accurate measurement of the propellant mass flow rate. Thus, all EP laboratory personnel must possess a cursory understanding of the working principles and uncertainties associated with MFCs. The purpose of this section is to consolidate the literature on MFCs, highlight the key physical phenomena governing MFC operation, and discuss the associated uncertainties inherent in MFC measurements.

Commercial MFCs are routinely used to measure and control the mass flow rate of gases into vacuum environments in industries ranging from semiconductor manufacturing to aerospace design and testing.[16] In EP applications, MFCs are often integrated into the propellant feed system and used to regulate the mass flow rate of propellant gas to the propellant distributor and external thermionic cathode.[17] Furthermore, MFCs are used to measure the propellant mass flow rate, which is a key parameter in the calculation of performance parameters such as propulsive efficiency and specific impulse as seen earlier. Few publications in the peer-reviewed literature include detail on theory of operation and uncertainties associated MFCs beyond listing the manufacturer, model number, and calibration accuracy.

There are two basic methods of flow rate control employed in modern MFCs: thermal mass flow control (TMFC) and pressure-based mass flow control (PMFC).[16,18] Thermal control relies on the transfer of heat from an electrically-heated wall to a moving fluid. Pressure control employs principles of compressible flow in order to measure and then regulate the mass flow rate of process gases. Because of these distinct differences, each type of MFC benefits from a separate analysis of its working principles and potential uncertainties.

A. Thermal Mass Flow Controllers

1. Working Principles and Operation

A diagram of a typical thermal mass flow controller system is shown in Fig. 1. For purposes of description, the internal components of the TMFC can be separated into two primary groups.[19] The first is the gas flow region shown in the bottom of the figure which consists of the inlet, turbulence filter, capillary bypass/flow sensor, valve, and outlet. As process gases enter the TMFC, a laminar flow restriction, shown in the figure as a decrease in the flow cross-sectional area, diverts a small percentage of gas through a capillary bypass tube to a thermal flow sensor and then back to the main flow region.[16,20] Note that the exact design of the thermal flow sensor and bypass differs between manufacturers. The components shown in Fig. 1 represent the only means by which a TMFC can determine the flow rate of the process gas.[21] Therefore, a thorough explanation of the working principles underlying the operation of these components is crucial to the understanding of the TMFC as a whole.

The capillary bypass is a thermally-conductive circular tube designed to ensure that fully developed laminar flow is present throughout the entire thermal sensor test section.[16] The prescription of laminar flow greatly simplifies the governing equations of heat transfer for the flow through the sensor, removes any potential turbulent transients that could otherwise cause error in the flow rate measurements, and allows for the tuning of the system to a particular Nusselt number based upon the thermal diffusion properties desired by the TMFC designer.[16]

The hardware composing the thermal sensor is equally simple and consists only of a set of heater coils wrapped around the capillary tube wall and attached to electrical control circuitry. While the exact number, location, and design of the coils varies based upon the manufacturer of the TMFC, the working principles governing their operation are identical and give rise to two distinct methods of mass flow rate measurement.[21]

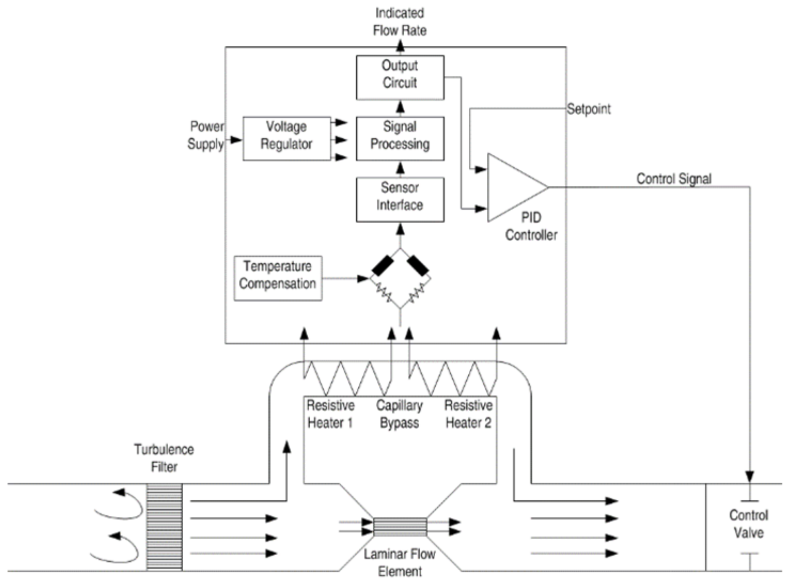

Fig. 1. Diagram of a Typical Thermal Mass Flow Controller.

In the first method, a constant power level is provided to the heater coils by the control circuitry and the temperature of the gas at the inlet and outlet of the tube is measured. Due to the motion of the gas, the temperature distribution along the tube is skewed such that the downstream temperature is larger than the upstream value, as depicted in Fig. 2.[21] By application of the first law of thermodynamics to the sensor region, the mass flow rate can be linearly related to the observed temperature differential using Eq. 5:

$$\dot{m} = \frac{\dot{Q}}{c_p(T)(T_o - T_i)} \tag{5}$$

where $c_p(T)$ is the temperature dependent heat capacity for the gas, $\dot{Q}$ is the rate of heat transfer from the capillary wall to the gas, T_o is the gas temperature at the outlet of the heated capillary, T_i is the gas temperature at the inlet of the heated capillary, and $\dot{m}$ is the mass flow rate.[21] The rate of heat transfer per unit length of the bypass in unit time required to maintain the observed temperature rise can be calculated using Eq. 6:

$$\frac{\dot{Q}}{L} = \pi a^2 \rho c_p u \frac{dT}{dx} \tag{6}$$

where $\frac{\dot{Q}}{L}$ is the rate of heat transfer per unit length of the pipe, a is the radius of the capillary, u is the mean gas flow velocity, ρ is the gas density, and T is the temperature of the pipe wall.[22]

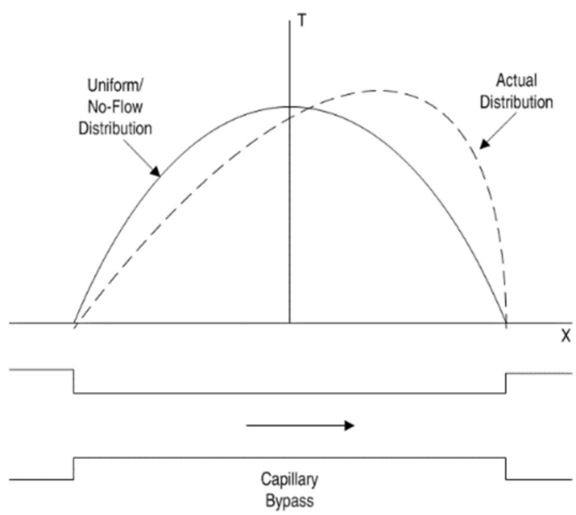

Fig. 2. Temperature Distribution in a Thermal Mass Flow Controller.

In the second method of measurement, a constant temperature level is maintained throughout the capillary tube independent of the mass flow rate. The power required to maintain a constant temperature is then related to the mass flow rate using King's law:

$$P_{heater} = P_{offset} + C\dot{m}^n \tag{7}$$

where P_{heater} is the power dissipated in the heater, P_{offset} is the offset heater power at no flow, C is a proportional constant used to take the heater dimensions and gas properties into account, and n is a dimensionless factor dependent on the Reynolds number (typically ~0.25).[23]

In both measurement methods, the heater signals are then passed to the second group of TMFC components, the electronics. In most cases the heaters are connected to electrical bridges and then fed into a PID controller circuit. The PID then compares the measured mass flow rate to the set point of the device and alters the position of the outlet valve to correct for any discrepancies.[20,23]

2. Uncertainties

In 1995, the National Institute of Standards and Technology (NIST) performed a thorough analysis of the potential sources of uncertainty for TMFCs. The study identified three general sources of uncertainty inherent to TMFC operation: calibration accuracy, reliance on gas correction factors (GCFs), and device susceptibility to changes in orientation, temperature, pressure, and mass flow rate.[21] As the occurrence of each of these errors is highly dependent upon the system in use, a separate evaluation of each is necessary in order to give the user a full and accurate picture of the errors that could be present in a given system of interest.

Most commercial TMFCs are calibrated by the manufacturer using nitrogen gas. Calibration accuracy refers to potential errors which may occur during this process and prevent the TMFC from operating within the manufacturer-specified accuracy range (typically ±1% full scale). Through independent calibration of several commercial TMFCs, NIST generally confirmed the accuracy of the manufacturer calibrations. However, this was not uniformly true as two of the tested TMFCs showed systematic differences ranging from -10% to 8.5% between the flow rate measured independently by NIST and the set-point of the TMFC. It was postulated that this difference is primarily due to differences in standards between the manufacturer and NIST, thus prompting the conclusion that this uncertainty can largely be mitigated through choice of manufacturer and device.[21]

Typically, manufacturers do not calibrate TMFCs for all possible process gases of interest.[23] As such, a relation known as the Gas Correction Factor (GCF) has been created in order to relate the ratio of the mass flow rate of the process gas and the mass flow rate of the calibration gas that will produce the same measurement.[21,23] In their analysis, NIST tested the accuracy of the manufacturer provided GCFs against independent measurements for argon, helium, sulfur hexafluoride, hexafluoroethane, and hydrogen. The results show that the accuracy of the GCF is highly dependent on the process gas of interest. Table 2 shows the average difference between manufacturer GCFs and measured GCFs for the gases and TMFCs used in the NIST investigation.

Table 2. Measured Errors in Gas Correction Factors (GCFs) for Selected TMFCs Using Various Process Gases.[21]

Process Gas	% Difference (Average – Manufacturer)
Ar	1.55
He	1.17
SF_6	-0.74
C_2F_6	7.70
H_2	10.22

The variation in the data in Table 2 between the process gases is most likely due to the relationship between temperature and specific heat capacity. As the internal temperature of the TMFC sensing element varies with the manufacturer, the exact heat capacity of the process gas flow is not easily determined. Furthermore, the reference values listed by manufacturers in manuals and used to compute GCFs are most often the value at 0 or 25 ºC while most sensors operate at temperatures of between 50-100 ºC. Gases with small heat capacity temperature coefficients, such as Ar and He, minimize the uncertainties associated with the GCF.[21]

The final source of uncertainty involves variations in TMFC zero and span due to device orientation, inlet pressure, inlet gas temperature, and low mass flow rates. The reliance of TMFC measurements on the heat transfer properties of the gas make the device susceptible to error whenever these properties are altered. As such, changing the orientation of the device such that the flow direction moves from perpendicular to parallel with the gravity vector can alter the zero point of the device by up to 0.4% of full scale with negligible changes observed for the device span. Similarly, variations in inlet pressure resulted in observed sensitivity changes of less than 0.1%, where manufacturer specifications typically indicate uncertainties of 0.75% per MPa of pressure change.[21]

The effects of temperature on the measurement can be separated into cases of heating of the electronics and heating of the process gas. For an increase of 10 ºC, both forms of heating show uncertainty in the zero point of the device of approximately 0.12% per degree above the calibration temperature. For the same temperature increase, alterations in the span varied with the type of heating and on average resulted in 0.80% per degree of full scale for heating of the electronics and 0.08% per degree of full scale for inlet temperature of nitrogen with noticeable variations occurring for different process gases.[21] Because of this variability, it is advisable to perform calibrations at the expected operational temperature of the MFC.

The uncertainties associated with stability and low mass flow rate are found to remain within the manufacturer specifications of ±1% of full scale.[21] A summary of all of the different operational uncertainties for TMFCs is shown in Table 3.

Table 3. Summary of operational uncertainties for TMFCs.[21]

Source	Effect	Measured Change (%FS)	Uncertainty (%FS)
Orientation	Zero	0.02-0.44	-
Orientation	Span	-	-
Upstream Pressure	Sensitivity	0.1	0.75 per MPa
Temperature	Zero	± 0.12	± 0.40-0.75
Temperature	Span	0.08 per ºC	0.75-1 per ºC
Stability	Zero	±0.04-0.4	-
Low Mass Flow Rate	Zero	±1	-

B. Pressure-Based Mass Flow Controllers

1. Working Principles and Operation

The increasing popularity of low-pressure-drop methods in the ion implantation community has resulted in the commercial development of mass flow controllers which utilize direct pressure measurement and regulation techniques in order to avoid the pressure losses of the process fluid associated with the thermal tube capillary bypass.[18] A diagram of a typical pressure-based mass flow controller is shown in Fig. 3.

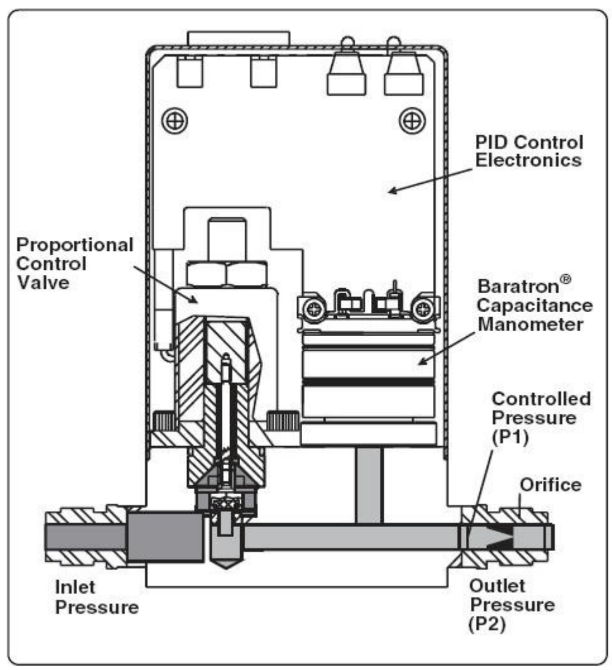

Fig. 3. Diagram of a Typical Pressure-Based Mass Flow Controller.

There are three primary PMFC measurement methods used to determine and regulate the mass flow rate through the device. The first two are known as the laminar flow and molecular flow methods and rely on differential pressure measurements across a flow element within the device in order to determine the mass flow rate.[18] These two methods regulate the flow by varying the valve position at the device outlet, which alters the internal pressure gradient and increases or decreases the mass flow rate.

The third PMFC measurement method is known as the critical (or sonic) flow method. This method utilizes the measurement and control of the pressure upstream of an orifice with a known area conductance to maintain choked flow conditions and determine flow rates. Specifically, referring to the notation in Fig. 3, the flow rate is proportional to P1 when P1 is at least twice the outlet pressure P2.[18] As such, P1 is varied by adjustment of the control valve position in order to precisely control the flow rate through the outlet and into the environment of interest. The flow rate through the orifice lies within the viscous flow regime, and, as such, it can be found using:

$$\dot{w} = C_o p A \sqrt{\left(\frac{\gamma R T}{M_w}\right)\left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}}} \tag{8}$$

where C_o is the orifice coefficient, p is the upstream pressure, and A is the orifice area.[24] Here the flow $\dot{w}$ is in units of pressure-volume and must be converted to mass flow rate for the specific working gas.

During operation, the process gas enters the inlet of the device and is regulated by the control valve orifice. The static pressure of the flow is measured with an internal manometer and the flow is accelerated to the sonic speed through an orifice before it reaches the exit the device. The manometer is connected to PID control circuitry which compares the measured pressure to the desired set-point and then varies the valve position until the error in the measurement is below the manufacturer-specified accuracy threshold.[18]

2. Uncertainty

Due to the use of a simpler, in-situ mass flow rate measurement technique, PMFCs should be susceptible to fewer sources of error as compared to TMFCs. Most noticeably, due to the direct control of inlet pressure in PMFCs, the uncertainty associated with inlet pressure should vanish while the removal of heat transport as a process of interest should limit the effects of temperature to electronic heating as opposed to uncertainties associated with the determination of the specific heat of the process gas.[18,21] Nevertheless, new sources of uncertainty associated with the included manometer must be evaluated with PMFCs. To date, no independent validation of PMFC uncertainty is yet available in the literature. Thus, manufacturer specifications are the best source of uncertainty data.

IV. Calibration

Mass flow meters must be routinely calibrated to achieve low uncertainties. Calibration can be either with a direct measurement of gas using NIST-traceable weights or a comparison to a NIST-traceable master flow device, but performance should be measured at least once a year in a procedure that defines accuracy and linearity. Instrument manufacturers generally can provide calibration servicing. Due to the cost and management overhead of maintaining calibration equipment and NIST-traceable standards (either the weights or the master flow device) most calibrations are handled by professional calibration laboratories, but a simple laboratory setup can enable users to do reference calibration checks using a secondary calibration method.

The first calibration method involves gravimetric weighing of the amount of gas that actually flows through the meter into or out of a container during the calibration procedure. This is normally done using a weigh scale that has a very high degree of accuracy. This weight scale must be approved by NIST or calibrated using standards traceable to NIST and be able to cover the entire range of possible mass unit outputs for that meter. The gravimetric method is generally regarded as the most accurate way to measure the actual mass flow rate.

The secondary method of calibration uses a master flow device in line with the MFC being tested to determine the behavior of the latter. The master flow device could either be a flow controller or a flow calibrator. Since this process relies on the master flow device, the device should be directly NIST-traceable by the manufacturer or professional calibration laboratory. This setup would allow a propulsion laboratory with a number of mass flow units to calibrate items by reference to a single unit rather than send all units out of house to a NIST-certified calibrator. This is a secondary calibration, so procedural controls should be implemented to account for possible calibration drift and to reduce the propagation of errors to other instruments. For example, secondary calibrations should be performed as close to the calibration of the master flow device as possible and no MFC should be referenced using the products of this procedure.

The calibration should ideally be performed with the MFC installed in the final configuration it will be used for EP testing, in order to avoid uncertainties associated with orientation and environmental conditions as noted earlier. The test setup should hold all units securely and contain a subset of flow system components for safe control (filter, gas regulator, on/off valves, etc.). A master flow controller would be installed in a location upstream of the tested MFC, and would be set at a commanded flow. The test flow is measured on the second unit, and a percent deviation is recorded as a percentage of difference over commanded flow:

$$\%\, deviation = \frac{Test\ Flow - Commanded\ Flow}{Commanded\ Flow} \times 100 \tag{9}$$

in which case percent deviations greater than 5% at any flow rate over the full scale could indicate possible damage or malfunction within the MFC. A master flow calibrator would be located downstream of the tested MFC and a similar deviation profile would be measured.

Calibration tests should be conducted for each specific gas to be used in the system to eliminate uncertainties associated with the use of gas correction factors. These should be done at the inlet pressure and environmental temperature expected during MFC use. For system use at a single or even a few mass flow rates a point calibration at those conditions may be appropriate. For broader application a point calibration becomes impractical, in which case the MFC should be calibrated across the intended flow scale range. Ideally, calibration checks should be performed at least at several points across the full scale range of the instrument. This process verifies linearity of the MFC as well as indicates some measure of reproducibility. Procedural steps should also be put into place to minimize deviations due to settling times and system contamination. All MFC calibration by comparison to a master flow device would have a minimum standard deviation based on the calibration of the master flow device.

V. Error Analysis and Uncertainty

An error in an experiment represents the difference between the measurement of a quantity and the true value of that quantity. An uncertainty, on the other hand, represents the range of values within which the true value is believed to exist. All measurements are subject to a degree of uncertainty and the characterization and quantification of the different sources of error help in determining the overall accuracy of the measurement. In this paper, a basic understanding of error analysis is assumed. For background information there are several excellent resources available (e.g. Taylor[25]).

Errors are typically classified as either random or systematic. Random errors are those that are omnipresent, unpredictable, and are scattered about the true value. Examples of systematic errors include a drift or offset in the zero value of an apparatus or the deformation over time of a scale used to measure length or volume. In flow measurements, especially those conducted over long periods of time, systematic errors can become an issue, affecting the zero value of the instrument or the coefficients in a function used to convert a measurement (for example, an output voltage) into the physical units of mass flow rate. For flow measurement, the method typically employed to handle systematic errors such as drift over time is to calibrate the apparatus. While calibration can help remove systematic errors, they cannot be completely eliminated because there are always intrinsic errors associated with any piece of hardware that would be used to calibrate the instrument. Various ambient conditions (such as temperature, for example) may cause additional, repeatable drifts or offsets that affect the overall measurement value. Such systematic errors can be characterized over an operating range, after which a monitoring of the ambient conditions can provide an additional calibration that can be used to correct for or partially remove the error.

A. Calibration Uncertainty

Most end-use instruments and sensors are delivered with NIST-traceable calibration data that specify the precision and uncertainty on the measurement. Calibration of individual units for the working gas and in the experimental setup for use in EP work, however, is recommended. Commercially-available calibration devices are available and routinely used for this and will often report an uncertainty associated with the flow measurement. Calibration is typically done using one of two common methods, either the constant volume or constant pressure method. A discussion of the sources of uncertainty for each method follows.

1. Constant Volume Calibration

A constant volume calibration is useful only for gases. This type of calibration is performed by flowing gas into a container of precisely known volume and monitoring the pressure in the container during the calibration process. The ideal gas equation of state is used in this constant-volume method:

$$pV = m\frac{R}{M_w}T \tag{10}$$

where V is the container volume and m is the mass of gas in the container. In practice, note that not all gases behave ideally and inclusion of a compressibility factor might be warranted in Eq. 10. If the process is also assumed to be isothermal, then the mass flow rate $\dot{m}$ into the volume is:

$$\dot{m} = \frac{dm}{dt} = \frac{M_w}{R}\frac{V}{T}\frac{dp}{dt} \tag{11}$$

The volume of the container is known to within an uncertainty σ_V, the temperature is constant to within a temperature range $\Delta T \approx \pm \sigma_T$, and the time-derivative of pressure has an error $\sigma_{dp/dt}$. The error in the mass flow rate is then given as:

$$\sigma_{\dot{m}} = \left[\left(\frac{M_w}{R} \frac{V}{T} \right)^2 \left(\sigma_{dp/dt} \right)^2 + \left(\frac{M_w}{R} \frac{V}{T^2} \frac{dp}{dt} \right)^2 \sigma_T^2 + \left(\frac{M_w}{R} \frac{1}{T} \frac{dp}{dt} \right)^2 \sigma_V^2 \right]^{1/2} \tag{12}$$

The terms on the right hand side give, respectively, the error contributions associated with the uncertainty on the pressure derivative, the 'constant' temperature, and the measurement volume being filled.

2. Constant Pressure Calibration

A constant pressure calibration is most useful for liquids, but it can also be used for gases. The calibration is performed by flowing gas into a container where the pressure is held constant while the contained fluid volume is measured and recorded as it changes with respect to time. For a gas, the ideal gas equation of state (Eq. 10) can again be used. Assuming a roughly isothermal process, the mass flow rate is:

$$\dot{m} = \frac{dm}{dt} = \frac{M_w}{R} \frac{p}{T} \frac{dV}{dt} \tag{13}$$

The error on the mass flow rate is estimated in a manner analogous to the previous example, and is written as:

$$\sigma_{\dot{m}} = \left[\left(\frac{M_w}{R} \frac{p}{T} \right)^2 \left(\sigma_{dV/dt} \right)^2 + \left(\frac{M_w}{R} \frac{p}{T^2} \frac{dV}{dt} \right)^2 \sigma_T^2 + \left(\frac{M_w}{R} \frac{1}{T} \frac{dV}{dt} \right)^2 \sigma_p^2 \right]^{1/2} \tag{14}$$

where the error on the pressure σ_p represents the degree to which the constant pressure assumption is correct.

For a liquid, we can assume incompressibility and obtain the mass flow rate through the continuity equation. Differentiating produces the very simple continuity relationship:

$$\dot{m} = \frac{dm}{dt} = \rho \frac{dV}{dt} \tag{15}$$

and the error is then:

$$\sigma_{\dot{m}} = \left[\rho^2 \left(\sigma_{dV/dt} \right)^2 + \left(\frac{dV}{dt} \right)^2 \sigma_\rho^2 \right]^{1/2} \tag{16}$$

where σ_ρ is the uncertainty in the incompressibility assumption and knowledge of the density at a given temperature.

3. Direct Weighing Measurement

The direct weighing calibration method is mainly useful for propellants that are stored as a solid or liquid and sublimated, ablated, or melted and vaporized within a thruster. Steady-state examples where this method may be useful include thrusters fed by water or liquid lithium, while a pulsed example is the ablative pulsed plasma thruster. If the propellant can only be weighed at the beginning and end of a test, then the average mass flow rate and error are easily determined. For a pulsed thruster, the time is replaced by the number of pulses to yield an average propellant mass expended per shot (mass bit).

In some cases, such as with liquid metals where electromagnetic flow sensors can be used,[5,6] flow through the sensor can be weighed directly in real time to correlate the output of the flow sensor with $\dot{m}$. In those instances, the flow through the sensor was deposited into a vessel located on a scale. In one method,[5] the mass output by the scale was converted using the fluid density to a time-history of the volume accumulated in the tank. An analytical function was fit to this time-history and then differentiated, yielding the volumetric flow rate, which was plotted as a function of the induced voltage in the flow sensor and curve fit by a line to yield the calibration coefficients to convert the output voltage to mass flow rate. In another method,[6] the data were used in the raw accumulated mass form, and numerically differentiated to yield the comparison between flow rate and flow sensor output voltage. In the reference, these data were also fit with a linear function to yield a calibration relating output voltage to mass flow rate. In both cases the error on the flow rate was taken as the error on the curve fit coefficients.

4. Time Derivatives in Calibration

In every method for flow calibration, a time-derivative of some measureable quantity must be obtained to calculate the flow rate. It is straightforward to calculate the uncertainty in a time-derivative if the measurement quantity is changing linearly with respect to time over a given time interval. Unfortunately, the linearity of a quantity over the specified time interval may not be known. A first recourse is to perform several measurements and examine the linearity. In cases where the linearity is not acceptable, it may be possible to reduce the time interval enough to achieve linearity without introducing larger sources of uncertainty. If that fails, other more sophisticated methods may be needed.

If the linear estimate is not an appropriate representation, and if enough data over the time interval Δt can be obtained, then it is possible that a different functional relationship can be used to relate the variable in question with time.[5] It is difficult to combine the errors on a series of individual data points and the errors on the curve fit. The error bar can be approximately estimated by curve fitting the data points and separately fitting the extremes of the data range dictated by the errors on the measurement and time. The error can be taken as the larger of the following: a) the maximum differences between the coefficients on the data point and extreme point curve fits or b) the error on the data point curve fit coefficients output by the fitting algorithm.

B. Other Uncertainties

Other environmental factors can affect the output of a flow meter, and care must be taken when high-precision measurements are desired or required. For example, changes to the ambient temperature can change the measurement of the gas flow rate.[26] The resulting errors can be either random or systematic, and each individual laboratory setup should be examined for sources of error. One way to handle this issue is to calibrate hardware under the expected operating conditions. Alternatively, calibration can be performed over a range of ambient conditions and then the associated environmentally-induced variation/uncertainty on the measurement can be accounted for in the overall error. If during the test the ambient conditions are being monitored, then the correct calibration for those conditions can be used to reduce the errors induced by changing conditions in the environment.

C. Curve Fitting Uncertainty

Flow meter calibration is typically performed by relating the output signal voltage of an MFC to the flow measured on a calibrating device for several different flow rates. This relationship is nominally linear depending on the quality of the MFC. For MFCs that will be used at a single, or even at a few operating points, it is feasible to perform point calibrations. For application to a broader range and number of flow rate set points this becomes unfeasible, however, and it is more convenient to construct a linear curve fit to the calibration data for determination of flow rates:

$$\dot{m} = A + Bv \tag{17}$$

where v is the output voltage of the MFC.

The coefficients A and B can easily be determined using a least-squares fitting method,[25] or alternatively can be performed by a number of commercially-available software programs. Because of the inherent uncertainty in the fitted data and the fact that they will rarely fall on a single line, the fit coefficients will also have uncertainties. Hence, the total uncertainty in mass flow rate calculated using Eq. 17 will be given by:

$$\sigma_{\dot{m}} = \sqrt{\sigma_A^2 + (v\sigma_B)^2 + (B\sigma_v)^2} \tag{18}$$

Caution must be exercised, however, when calculating these fit coefficient uncertainties or using the values calculated by curve-fitting software. The standard method of calculating these for data pairs of $(v_1, \dot{m}_1), \ldots, (v_N, \dot{m}_N)$ where the uncertainty in the v_i data are negligible is:[25]

$$\sigma_A^2 = \frac{\sigma_{\dot{m}}^2 \sum v_i^2}{N(\sum v_i^2) - (\sum v_i)^2} \tag{19}$$

$$\sigma_B^2 = \frac{N\sigma_{\dot{m}}^2}{N(\sum v_i^2) - (\sum v_i)^2} \tag{20}$$

Notice the dependence for each equation is on the number of measurements *N*, the measured $\dot{m}_i$ values, and the uncertainty in the parameter $\dot{m}$. Without knowledge of the real uncertainty in the measurement of the values $\dot{m}_i$, a packaged software program will likely determine a value from the distribution of $\dot{m}_i$ values compared to the linear fit. This ignores the knowledge of the uncertainty calculated with Eqs. 12 or 14, for example, and can be a poor representation of the actual uncertainty in the measurement. Hence, the value of the flow rate uncertainty calculated with Eq. 12 or 14, or that provided by a commercial calibration device, should be used in conjunction with Eqs. 19 and 20 to determine the fit coefficient uncertainties, σ_A and σ_B. These should in turn be used in Eq. 18 to determine the total flow rate uncertainty when using a linear curve fit to the calibration data. If the uncertainty in voltage measurement is appreciable compared to the flow measurement uncertainty then more rigorous methods may be necessary.

VI. Conclusion and Recommendations

Accurate control and measurement of propellant mass flow rate to a thruster is one of the most basic and fundamental requirements for EP systems, whether they be in the laboratory or on flight spacecraft. Hence, it is important for the EP community to have a common understanding of and methods for flow control and measurement. The material presented here focuses mainly on gaseous propellant control and measurement which is a well-developed field. Propellant flow measurement for systems that do not utilize gaseous propellants is largely a developing area.

Most EP laboratories use commercially available mass flow controllers or meters based on either thermal- or pressure-sensing technology, which fortunately are relatively inexpensive and easy to use. Thermal MFCs monitor flow by inducing a temperature change in a flowing gas. These devices are susceptible to uncertainties associated with the use of gas correction factors, and changes in orientation, temperature, pressure, and mass flow rate, due to their reliance on the heat transfer properties of the gas. Pressure-based MFCs monitor flow by measuring a differential pressure across a flow element or the pressure upstream of a sonic orifice. These are less susceptible to inlet pressure and gas heat transfer property uncertainties than the thermal MFCs, but introduce additional uncertainties related to internal pressure measurement.

The vast majority of EP flight systems control propellant flow by controlling the pressure upstream of a sonic orifice. In the case of ion thruster systems these orifices are calibrated prior to launch and the in-flight pressures are measured so that the flow rates are known. Calibration is not required for Hall thruster systems that control flow based on discharge current feedback and thus do not need to measure the upstream pressure. Hence, flow rates for Hall thruster systems are not determined from flight telemetry (but could be with the inclusion of pressure transducers and orifice calibration). Remaining propellant is calculated using bookkeeping, pressure-volume-temperature, and/or thermal mass gauging methods.

Accurate and regular calibration of MFCs is critical for laboratory use and in the development of flight systems. Calibrations should be traceable to NIST and performed at least every twelve months over the entire flow scale. In addition it is highly desirable that the calibrations be performed with the working gas under study instead of relying on a gas correction factor, and in the orientation and environment that they will be used for system testing. MFCs can be sent out to vendors for calibration, or preferably this can be accomplished in the final test configuration with a commercially-available calibration unit. Accurate constant-volume and constant-pressure calibrators are widely available. Procedural steps should be put into place that minimize deviations due to settling times, system contaminations, and environmental changes during periods of calibration.

Methods for determining the calibration uncertainty for constant-pressure and constant-volume methods were also discussed. General texts in error analysis should be consulted for background information and basic methods. For EP testing conducted at a single flow rate a calibration can be performed at that point, yielding the best estimate of flow uncertainty. For larger flow ranges a linear curve fit is typically performed with calibration data which introduces additional uncertainty in reported flows. A method for determining this additional uncertainty was also discussed.

Due to the fundamental nature of propellant flow in the performance and characterization of EP systems, it is important that technical presentations and community discussions always address flow control methods when discussing the propulsion system. It is recommended that any reporting of thruster test data include a description of the type of technology used for flow control/measurement and the calibration status of the flow control device (i.e. NIST-traceable calibration performed with the last twelve months). For reports including thruster performance or measurements specific to gas or plume physics, the results of a careful uncertainty analysis of the flow measurement methods and possible impacts on the reported metrics should be included.

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